King Fahd University of Petroleum and Minerals
College of Computing and Mathematics
Department of Mathematics

MATH 513 - Final Exam

AY 2024-2025 (Term 241)

Time Allowed: 120 Minutes

Name: .... **ID number:** ....

- Textbook, notes, mobiles and smart devices are not allowed in this exam.
- Write neatly and legibly. You may lose points for messy work.
- Show all your work. No points for answers without justification.

Question	**Marks**
1	
2	
Total	

MCQ	**Answer**
3	
4	
5	
6	
7	
8	
Total	

Question 1

Use the separation of variables method to solve the Laplace equation:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < L,\, 0 < y < H,$$

subject to:

$$u(0, y) = 0, \quad u(L, y) = 0, \quad u(x, 0) = 0, \quad u(x, H) = f(x),$$

where $f(x)$ is a given function.

Question 2

Consider the initial boundary value problem (IBVP):

$$\begin{cases} u_t = 7u_{xx}, & \text{for } 0 < x < 5, t > 0 \\ u(0,t) = 1, \; u(5,t) = 4, & \text{for } t > 0 \\ u(x,0) = x + 1, & \text{for } 0 < x < 5. \end{cases} \tag{1}$$

Assume $u(x,t) = v(x,t) + \psi(x)$, where

$$v(x,t) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{5}\right) e^{-7n^2\pi^2 t/25}$$

satisfies the heat equation $v_t = 7v_{xx}$ for $0 < x < 5, t > 0$ and $v(0,t) = v(5,t) = 0$.
Solve the IBVP (1) by finding $\psi(x), v(x,0)$ and c_n.

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Question 3

Consider the wave equation: $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}, \quad -\infty < x < \infty,\, t > 0,$
with the initial conditions: $u(x,0) = H(x-1), \quad \frac{\partial u}{\partial t}(x,0) = \frac{1}{x^2+1}$
Then, $u(x,t) =$

(A) $\frac{1}{2}\left[H(x+2t-1) + H(x-2t-1)\right] + \frac{1}{4}\left[\tan^{-1}(x+2t) - \tan^{-1}(x-2t)\right]$

(B) $\frac{1}{4}\left[\tan^{-1}(x+2t) + \tan^{-1}(x-2t)\right]$

(C) $\frac{1}{2}\left[H(x+t-1) - H(x-t+1)\right] + \frac{1}{2}\left[\tan^{-1}(x+t) - \tan^{-1}(x-t)\right]$

(D) $\frac{1}{2}\left[H(x+4t-1) + H(x-4t+1)\right] + \frac{1}{2}\left[\tan^{-1}(x+4t) + \tan^{-1}(x-4t)\right]$

(E) None of the above.

Question 4

The function $f(t)$ is defined as:

$$f(t) = \begin{cases} e^{2t}, & t < 0, \\ e^{-t}, & t > 0. \end{cases}$$

The Fourier Transform $F(\omega)$ of $f(t)$ is

(A) $\frac{3}{(2-i\omega)(1+i\omega)}$

(B) $\frac{2}{(2+i\omega)(1-i\omega)}$

(C) $\frac{1}{(1+i\omega)^2}$

(D) $\frac{1}{(2-i\omega)^2}$

(E) None of the above.

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Question 5

Using Parseval's equality and the fact that $\mathcal{F}\left\{\frac{1}{1+t^2}\right\} = \pi e^{-|\omega|}$, $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^2} =$

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{8}$

(D) $\frac{\pi}{6}$

(E) None of the above.

Question 6

Find the Laplace Transform of $f(t) = 2\sin(t) - \cos(2t) + \cos(3)$.

(A) $\frac{2}{s^2+1} - \frac{s}{s^2+4} + \frac{\cos(3)}{s}$

(B) $\frac{2s}{s^2+1} - \frac{s}{s^2+4} + \frac{\cos(3)}{s} - \frac{1}{s^2}$

(C) $\frac{2}{s^2+1} - \frac{s}{s^2+4} + \frac{\cos(3)}{s^2}$

(D) $\frac{2}{s^2+1} - \frac{s}{s^2+4} + \frac{s}{s^2+9}$

(E) None of the above.

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Question 7

The inverse Laplace Transform of $F(s) = \frac{2s+3}{s^2+9}$.

(A) $2\cos(3t) + \sin(3t)$

(B) $2\sin(3t) + \cos(3t)$

(C) $2\cos(3t) + 3\sin(3t)$

(D) $2\sin(3t) - \cos(3t)$

(E) None of the above.

Question 8

What is the Laplace transform $Y(s)$ of the solution to the initial value problem $y'' + y = t$, with initial conditions $y(0) = 1$ and $y'(0) = 0$?

(A) $\frac{s^3+1}{s^4+s^2}$

(B) $\frac{s+1}{s^2+1}$

(C) $\frac{s^2+1}{s^3+s}$

(D) $\frac{s^2+s}{s^3+1}$

(E) None of the above.