King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Major Exam 1
Term (251)

Time Allowed: 120 Minutes

Name: _____ ID#: _____

Instructor: _____ Sec #: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		15
2		15
3		30
4		20
5		20
Total		100

Q:1 (15 points) Solve the following system:

$$x_1' = 2x_1 - 3x_2$$
$$x_2' = 3x_1 + 2x_2,$$

where the primes denote the time derivative.

Q:2 (15 points) Find e^{At} for the matrix $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.

Q:3 (30 points)

(a) Determine the Fourier series $S(t)$ of the even function $f(t) = t^2$ over the interval $[-\pi, \pi]$.

(b) By making use of the result obtained in part (a), evaluate the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^4} \ .$$

(c) Discuss whether the Fourier series of $g(t) = t$ can be derived from $S(t)$. If this is possible, explicitly compute it.

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Q:4 (20 points) Find the complex Fourier series of $f(t) = \begin{cases} 1, & 0 < t < \pi, \\ -1, & -\pi < t < 0. \end{cases}$.

Q:5 (20 points) Find the eigenvalues and eigenfunctions of the following boundary value problem

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(\pi) = 0.$$

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