King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Major Exam 2
(251)

Time Allowed: 120 Minutes

Name: _____ ID#: _____

Serial #: _____

- All types of electronic devices such as programmable calculators, smart watches or mobile phones are NOT allowed during the examination.
- Justify your answers, No credit is given for (correct) answers not supported by work.
- Write neat and clean your answers.
- Students are not allowed to take the exam without KFUPM ID card or national ID card or Iqama or Passport.

Question #	Marks	Maximum Marks
1		30
2		10
3		30
4		30
Total		100

Q:1 (30 points) Solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} + \gamma u = c^2 \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < \pi, \quad 0 < t,$$

where $\gamma > 0$ is constant, subject to the boundary conditions

$$\frac{\partial u(0,t)}{\partial x} = \frac{\partial u(\pi,t)}{\partial x} = 0, \qquad 0 < t,$$

and the initial conditions

$$u(x,0) = 0, \qquad \frac{\partial u(x,0)}{\partial t} = 1 + \cos(x), \qquad 0 < x < \pi.$$

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Q:2 (10 points)Use D'Alembert formula to solve the following wave equation

$$c^2 u_{xx} = u_{tt}, \quad -\infty < x < \infty, \quad t > 0,$$

subject to the initial conditions

$$u(x,0) = \cos(3x), \quad u_t(x,0) = \sin^2(x), \quad -\infty < x < \infty.$$

Q:3 (30 points) Solve the heat equation

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < \pi, \quad 0 < t,$$

where

$$u(0,t) = 0, \qquad u(\pi,t) = \beta_0, \qquad u(x,0) = \beta_0.$$

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Q:4 (30 points) Solve the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < \pi, \quad 0 < y < 1,$$

along with the boundary conditions

$$u(x,1) = 1 + x,$$
$$u_x(0,y) = u_x(\pi,y) = 0, \qquad \text{and} \qquad u_y(x,0) = 0.$$

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