King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Final Exam
The First Semester of 2025-2026 (251)

Time Allowed: 180 Minutes

Name: ____________________________ ID#: ______________

Serial #: ________

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear to get full credit.

Question #	Marks	Maximum Marks
1		20
2		20
3		20
4		20
5		20
6		20
7		20
8		20
Total		160

Q:1 (20 points) Solve the following ordinary differential equation by Fourier series if the forcing is given by the periodic function

$$f(t) = \begin{cases} 1, & 0 < t < \pi, \\ 0, & \pi < t < 2\pi, \end{cases} \qquad \text{and } f(t) = f(t + 2\pi) :$$

$$y'' - 3y' + 2y = f(t).$$

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Q:2 (20 points) Find the eigenvalues and eigenfunctions of the following differential equation

$$\frac{d}{dx}\left(x^3 y'\right) + \lambda x y = 0, \quad y(1) = y(e^\pi) = 0, \quad 1 \le x \le e^\pi.$$

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Q:3 (20 points) Solve the following heat equation

$$\frac{\partial u}{\partial t} - a^2 \frac{\partial^2 u}{\partial x^2} = e^{-x}, \qquad 0 < x < \pi, \quad 0 < t,$$

subject to the boundary conditions

$$u(0,t) = -\frac{1}{a^2}, \quad u_x(\pi, t) = 0, \qquad 0 < t,$$

and the initial condition

$$u(x,0) = -\frac{1}{a^2} e^{-x}, \qquad 0 < x < \pi.$$

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Q:4 (20 points) Use direct integration to find the inverse of the Fourier transform

$$F(\omega) = \sqrt{\frac{\pi}{a}}\,\exp\left(-\frac{\omega^2}{4a}\right).$$

Q:5 (20 points) Use the function $f(t) = e^{-at}\sin(bt)\,H(t)$ with $a > 0$ and Parseval's equality to show that

$$2\int_0^\infty \frac{dx}{(x^2+a^2-b^2)^2+4a^2b^2} = \int_{-\infty}^\infty \frac{dx}{(x^2+a^2-b^2)^2+4a^2b^2} = \frac{\pi}{2a(a^2+b^2)}.$$

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Q:6 (20 points) Verify the Fourier convolution theorem of the functions

$$f(t) = H(t + a) - H(t - a) \quad \text{and} \quad g(t) = e^{-t} H(t), \qquad a > 0.$$

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Q:7 (20 points) Solve the following integral equation

$$f(t) = 6t + 4\int_0^t f(x)\,(x-t)^2\,dx$$

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Q:8 (20 points) Use Laplace transform method to solve the wave equation

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = te^{-x}, \qquad 0 < x < \infty,\ 0 < t,$$

with the boundary conditions

$$u(0,t) = 1 - e^{-t}, \qquad \lim_{x\to\infty} |u(x,t)| \sim x^n,\ n \text{ finite},\ 0 < t,$$

and the initial conditions

$$u(x,0) = 0, \qquad u_t(x,0) = x, \qquad 0 < x < \infty.$$

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