King Fahd University of Petroleum and Minerals Department of Mathematics Math 525 Exam II Fall 2024(T241)

ID#:	NAME:	Total Score $\#$

Instructions

Answer all questions in detail. Provide proofs or counterexamples where requested. Clearly state definitions and theorems as provided in the course. Use proper notation and justify all your answers.

- 1. Define the following terms:
 - Planar Graph
 - Dual Graph
 - A k-edge-cut
 - *k*-Vertex-Colourable Graph
 - *k*-Connected Graph

- An S-component of a connected graph G
- 2. State the following theorems:
 - Euler's Formula for planar graphs
 - Five-Colour Theorem
 - Four-Colour Theorem
 - Tait's Theorem
 - Menger's Theorem on connectivity (include both the vertex and edge versions) and define all needed concepts used to state the theorems
 - Fan Lemma
 - Characterization of a chordal graph

- 3. Prove or disprove the following statements. Clearly state whether you are proving or providing a counterexample:
 - If G is a connected graph with all degrees even, then G is 2-edge-connected.

• A graph G on at least k+1 vertices is k-connected if and only if G-X is connected for every vertex set X of size k-1.

• All 3-connected planar graphs have a unique dual graph.

• Every bipartite graph is 2-edge-colorable.

4. Show how Euler's formula can be used to prove that neither K_5 nor $K_{3,3}$ is planar. Provide a detailed solution.

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