MATH 528 MIDTERM EXAM (TERM 211)

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NAME:

1. Let V be the vector space of degree at most 2 polynomials, i.e.

$$V = \{f(x) = a_0 + a_1 x + a_2 x^2 : a_1, a_2, a_3 \in \mathbb{R}\}.$$

Let $T : V \to V$ be the linear transform defined by

$$Tf(x) = xf'(x) + x^2 f''(x).$$

Find the matrix representation $[T]_{\mathcal{B}}$, where $\mathcal{B} = \{1, x, x^2\}$.

2. Find eigenvalues and corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 4 \end{bmatrix}.$$

3. Check that

$$\alpha(s) = \left(\frac{\cos^{-1} s - s\sqrt{1-s^2}}{2}, \frac{1-s^2}{2} \right)$$

is a unit speed curve. Find $\vec{T}(s)$, $\vec{N}(s)$ and the curvature $\kappa(s)$. (Hint. Use $\frac{d}{ds}(\cos^{-1} s) = -\frac{1}{\sqrt{1-s^2}}$.)

4. Compute the curvature $\kappa(t)$ and the torsion $\tau(t)$ of the curve

$$\alpha(t) = (6t, 3t^2, t^3).$$

5. Check

$$\mathbf{x}(u, v) := (\sin u \cos v, 2 \sin u \sin v, 4 \cos u)$$

is a parametric surface.

6. Consider the Monge patch

$$\mathbf{x}(u, v) = (u, v, u^2 + v^2).$$

Compute $\mathbf{x}_u$ and $\mathbf{x}_v$ at $(u, v) = (2, 1)$. Find also a and b such that

$$\vec{v} = (-1, 2, 0) = a\mathbf{x}_u + b\mathbf{x}_v$$

at $(u, v) = (2, 1)$. (This means $\vec{v}$ is a tangent vector of the surface at $(2, 1, 5)$.)