MATH 528 FINAL EXAM (TERM 211)

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1. Let $P_2 = \{f(x) = a_0 + a_1 x + a_2 x^2 : a_0, a_1, a_2 \in \mathbb{R}\}$ be the vector space consisting of polynomials of degreee ≤ 2. For $f, g \in P_2$. the inner product $\langle f, g\rangle$ is defined by

$$\langle f, g\rangle := \int_0^1 f(x)g(x)\,dx.$$

Let $Q_2 = \{f(x) = a_0 + a_2 x^2 : a_0, a_2 \in \mathbb{R}\}$ be the subspace of P_2 consisting of polynomials without the first degree term.

(a) Find $a > 0$ and b such that

$$f_1 \equiv 1, \quad f_2(x) = a + bx^2$$

form an orthonomal basis for Q_2.

(b) Find the orthogonal projection of $g(x) = x \in P_2$ onto Q_2.

2. Compute the Gaussian and the mean curvature of the Monge patch

$$\mathbf{x}(u, v) = (u,\ v,\ e^{2u-v})$$

at $(u, v) = (0, 0)$.

3. Compute the Gaussian and the mean curvatures of

$$\mathbf{x}(u, v) = (2u^2 - v^2 - u,\ u^2 + 2v^2,\ u + v)$$

at $(u, v) = (0, 0)$.

4. Compute the Gaussian and the mean curvatures of

$$\mathbf{x}(u, v) = (v \cos u,\ v \sin u,\ e^u)$$

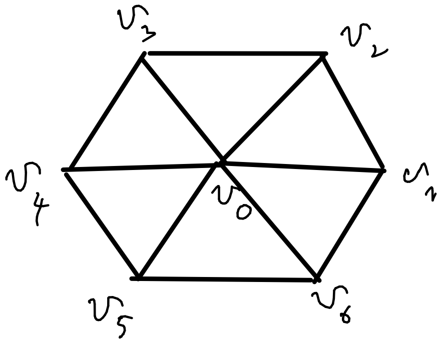
at $(u, v) = (0, 0)$.

5. Compute the principal curvatures and principal directions of the surface of revolution

$$\mathbf{x}(u, v) = ((u^2 + 1)\cos v,\ (u^2 + 1)\sin v,\ u)$$

at $(u, v) = (0, 0)$.

6. Suppose that the 1-ring of $v_0 = (0, 0, 0)$ in a triangular mesh is hexagonal shape as below.



Compute the discrete Gaussain curvature at v_0 in case $v_1 = (0.2,\ 0,\ 0.04)$, $v_2 = (0.1,\ 0.2,\ -0.03)$, $v_3 = (-0.1,\ 0.2,\ -0.03)$, $v_4 = (-0.2,\ 0,\ 0.04)$, $v_5 = (-0.1,\ -0.2,\ -0.03)$, and $v_6 = (0.1,\ -0.2,\ -0.03)$.