جامعة الملك فهد للبترول والمعادن
King Fahd University of Petroleum & Minerals

College of Computing and Mathematics
Department of Mathematics

MATH531 – Real Analysis

Major Exam 1
Term 222

February 26, 2023

Time allowed: 110 Minutes.

Name	
Student ID #	

Question #	Mark	Maximum Mark
1		15
2		15
3		10
4		13
5		14
6		19
7		14
Total		100

Instruction: Give the details of every solution (proof).

Exercise 1

(a) If E_1 is a finite set, show that $m^*(E_1) = 0$.
(b) Let A be the set of irrational numbers in the interval $[0, 1]$. Prove that $m^*(A) = 1$.

Exercise 2 Let E have finite outer measure. Show that E is measurable if and only if for each open, bounded interval (a, b),

$$b - a = m^*\big((a, b) \cap E\big) + m^*\big((a, b) \sim E\big).$$

Exercise 3 (*Solve only one of the following problems*)

(a) Show that if a set E has positive outer measure, then there is a bounded subset of E that also has positive outer measure.
(b) Let E have finite outer measure. Show that if E is not measurable, then there is an open set $\mathcal{O}$ containing E that has finite outer measure and for which

$$m^*(\mathcal{O} \sim E) > m^*(\mathcal{O}) - m^*(E).$$

Exercise 4 Let f be a measurable function on E that is finite a.e. on E and $m(E) < \infty$. For each $\varepsilon > 0$, show that there is a measurable set F contained in E such that f is bounded on F and $m(E \sim F) < \varepsilon$.

Exercise 5 State the three Littlewood principles and state the three related theorems (precise formulations of the Littlewood principles).

Exercise 6 (*Solve <u>two</u> of the following problems including (c)*)

(a) Prove that the extension of Lusin's Theorem to the case that f is finite a.e.
(b) Show that the conclusion of Egoroff's Theorem can fail if we drop the assumption that the domain has finite measure.
(c) Suppose a function f has a measurable domain and is continuous except at a finite number of points. Show that f is measurable.

Exercise 7 Let f be a nonnegative bounded measurable function on a set of finite measure E. If $\int_E f = 0$, show that $f = 0$ a.e. on E.