Department of Mathematics, King Fahd University of Petroleum & Minerals,
Math 533 Exam-02, 2024-2025 (241)
Time Allowed: 120 Minutes

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles, calculators and smart devices are not allowed in this exam.
- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. **No points for answers without justification.**

Question #	Marks	Maximum Marks
1		15
2		15
3		15
4		15
5		15
Total		75

Q1: (a) State and prove Liouville's theorem using Cauchy's integral theorem (do not use Cauchy's inequality).

OR

Q1: (a) State and prove Moreara's theorem.
(b) Prove that for any entire function $f(z)$ such that $Re(f(z)) \geq Im(f(z))$ for all $z \in C$ is constant.

Q2: (a) State Cauchy's inequality. Let $f(z) = \dfrac{1}{(1-z)^2}$, $0 < R < 1$. Verify that $\max\limits_{|z|=R} = \dfrac{1}{(1-R)^2}$. Also show that $f^{(n)}(0) = (n+1)!$ and $(n+1)! \leq \dfrac{n!}{R^n(1-R)^2}$.
(b) Evaluate $\int_C \dfrac{Log(z)}{2z^2 - 2z + 1} dz, \quad C : \{z : |z-1| = \dfrac{8}{9}\}$.

Q3: (a) State Gauss's Mean-Value theorem and use it to evaluate

$$\int_o^{2\pi} Log(b^2 + 1 + 2b\cos(\mathrm{n}\theta))\mathrm{d}\theta.$$

(b) State Maximum Modulus theorem and use it to find the maximum value of $|z^2+2z-3|_{|z|\leq 1}$.

Q4: (a) Expand $f(z) = \dfrac{z^2}{z^2 - 3z + 2}$ in a Laurent series valid for $1 < |z-3| < 2$.
(b) Let $f(z)$ be an entire function satisfying that $|f(z)| \leq |z|^2$ for all z. Show that $f(z) \equiv az^2$ for some constant a satisfying $|a| < 1$.

Q5: (a) Find a bilinear transformation that maps the points $z = 1-i, 1+i, -1+i$ onto $w = 0, 1, \infty$.
(b) Find the image of the ∞ strip $\frac{1}{4} \leq y \leq \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. Also sketch the images.