Department of Mathematics, King Fahd University of Petroleum & Minerals,
Math 533 Exam-01, 2025-2026 (251)

Time Allowed: 120 Minutes

Name: ______________________ ID#: ______________

Section/Instructor: ______________ Serial #: ______________

- Mobiles, calculators and smart devices are not allowed in this exam.
- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. **No points for answers without justification.**

Question #	Marks	Maximum Marks
1		20
2		20
3		30
4		30
Total		100

Q1: **(a)** Show that there are complex numbers z satisfying

$$|z-a|+|z+a|=2|c|$$

if and only if $|a|\leq|c|$. If this condition is fulfilled, what are the smallest and largest values of $|z|$?

(b) Solve $(z+2)^3=3i$.

(c) Find the radius of convergence of the following power series

$$(i)\quad \sum_{n=0}^{\infty}\frac{z^n}{n!} \qquad\qquad (ii)\quad \sum_{n=0}^{\infty}\frac{1}{(1+i)^n}\,z^{3n}$$

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Q2: Let $f(z)$ be analytic in a domain D and prove that f is constant if it satisfies any of the following conditions:

(a) $|f(z)|$ is constant.

(b) $\Re(f(z))$ is constant.

(c) $\arg(f(z))$ is constant.

(d) $\overline{f(z)}$ is analytic.

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Q3: (a) Let $f(z) = u + iv$ be analytic in a domain D and suppose

$$u + v = e^x(\cos y + \sin y).$$

Find $f(z)$ in terms of z.

(b) Find all complex numbers z satisfying

$$z^{1-i} = -4.$$

(c) For what values of $z \in \mathbb{C}$ is the series

$$\sum_{n=0}^{\infty} \left(\frac{z}{1+z} \right)^n$$

convergent?

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Q4 (a) Use ML-inequality to find the upper bound of $\int_C \frac{z}{z^2+1}\,dz$, C: straight line from 2 to 2+i.

(b) Define the function

$$f(z) = \begin{cases} \dfrac{z\,\Re(z)}{|z|}, & z \neq 0, \\ 0, & z = 0. \end{cases}$$

Prove that $f(z)$ is continuous in the entire complex plane.

(c) Write the parametric equation of the contour $C = C_1 + C_2$, where C_1 is the circular arc in the first quadrant joining 4 and $4i$, and C_2 is the line segment in the second quadrant joining $4i$ and -4.