Department of Mathematics, King Fahd University of Petroleum & Minerals,
Math 533 Exam-02, 2025-2026 (251)

Time Allowed: 110 Minutes

Name: ____________________ ID#: ____________________

Section/Instructor: ____________________ Serial #: ____________________

- Mobiles, calculators and smart devices are not allowed in this exam.
- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. **No points for answers without justification.**

Question #	Marks	Maximum Marks
1		15
2		25
3		30
4		30
Total		100

Q1: (a) Let $f(z)$ be a continuous function on a domain D and suppose that $\int_C f(z)\,dz = 0$ for every closed contour C lying entirely in D. Prove that $f(z)$ has an antiderivative throughout D.

(b) Let $f(z) = \dfrac{1}{z}$ be defined on the domain $D = \mathbb{C} \setminus \{0\}$.

(i) Show that $\displaystyle\int_C f(z)\,dz \neq 0$ for some closed contour C in D.

(ii) Explain why $f(z)$ does not have an antiderivative throughout D.

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Q2: (a) State and prove Morera's theorem.

(b) Let $f(z) = \overline{z}^2$. Evaluate $\oint_{|z|=1} f(z)\, dz$ and verify Morera's condition.

(c) Show that an entire function whose real part is nonpositive is constant.

(d) State Gauss's Mean-Value theorem and use it to evaluate

$$\int_{-\pi}^{\pi} \cos^2\left(\frac{\pi}{6} + ae^{i\theta}\right)\, d\theta.$$

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Q3:(a) Find the Laurent series expansion of

$$f(z) = \frac{z^2+1}{z^2(z-2)}$$

in the regions (i) $0 < |z| < 2$ and (ii) $|z| > 2$

(b) Let f be analytic inside and on the unit circle. Suppose that

$$|\, f(z) - z \,| < |z| \quad \text{for } |z| = 1.$$

Show that $|f'(1/2)| \leq 8$.

(c) Find the maximum value of

$$\left|(z-2)\left(z+\tfrac{1}{3}\right)\right|$$

for all complex numbers z satisfying $|z| \leq 1$.

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Q4: (a) Suppose f is analytic for $|z| < 2$ and $\alpha \in \mathbb{C}$ is constant. Evaluate

$$I = \oint_{|z|=1} \big(\operatorname{Re} z + \alpha\big) \frac{f(z)}{z}\, dz.$$

(b) Let f be entire and suppose

$$|f(z)| \le 5 + 0.01\,|z| \quad (\forall z \in \mathbb{C}).$$

(i) Show that $f(z) = az + b$ with $|a| \le 0.01$, $|b| \le 5$.

(ii) If additionally $f(0) = 2$ and $f(100) = 3$, find $f(z)$.

(c) Evaluate $\displaystyle\oint_{|z-2|=3} \frac{dz}{z^2+1}$.

(d) Show that there does not exist an entire function $f(z)$ such that

$$|f(z)| < 5 \quad \text{for all } z \in \mathbb{C},$$

and

$$f(0) = 2, \qquad f(1) = 3.$$

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