EXAM 2

Duration: 120 minutes

ID:	
NAME:	

- Show your work.
- There are empty pages attached to this exam booklet.

Problem	Score
1	
2	
3	
4	
5	
Total	/100

Problem 1 (20 points)

The vector space U is endowed with a semi-norm p. A relation $u \sim v$ on U where $u, v \in U$is defined by $p(u - v) = 0$. Show that this relation is an equivalence relation on U and the equivalence class $[0]$ is a subspace of U.

Problem 2 (20 points)

Consider a normed space U.

(a) Prove that $\overline{\mathfrak{M}}$ is a subspace of U, where $\mathfrak{M}$ is a subspace of U.

(b) If $\mathfrak{N}$ is nonempty subset of U, show that the closed linear hull of $\mathfrak{N}$ is the closure of the linear hull of $\mathfrak{N}$.

Problem 3 (20 points)

Let $(U, \|\cdot\|_1)$ and $(U, \|\cdot\|_2)$ be two Banach spaces on the same vector space U. Assume that there exists a constant $\mu > 0$ such that

$$\|u\|_2 \leq \mu \|u\|_1 \qquad \text{for all } u \in U.$$

Show that $\|\cdot\|_1$ and $\|\cdot\|_2$ are equivalent.

Problem 4 (20 points)

Let $\mathfrak{M}$ be a closed subspace of a reflexive Banach space U. Show that $\mathfrak{M}$ is also reflexive, i.e $\mathfrak{M}'' = \mathfrak{M}$.

Problem 5 (20 points)

Let U and V be two normed spaces, $A : U \to V$ be a continuous linear operator and A' its conjugate. Assume that A^{-1} exists and continuous. Show that for every $f \in U'$, there exists $g \in V'$ such that $f = A'g$.