Math 550 - Exam I

150 minutes

Max Score: 50

Name:

ID:

Q1 [12 pts] Prove the following theorems:

1) **[6]** Let W_1 and W_2 be finite-dimensional subspaces of a vector space V. Then the subspace $W_1 + W_2$ is finite-dimensional and:

$$\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2)$$

2) **[6]** The rank-nullity theorem. (State the theorem, then prove it.)

Q2 [14 pts]. Consider the linear transformation:

$$T : \mathbb{R}^2 \to \mathbb{R}^3, \quad T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+y \\ x-2y \\ 3x \end{bmatrix}$$

1) **[6]** Find $[T]_{B_1}^{B_2}$, where $B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$, and $B_2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

2) **[2]** Find $\ker T$. Is T an isomorphism? (Justify your answer.)

3) **[6]** Let f be a linear functional on $\mathbb{R}^3$ defined by $f \begin{bmatrix} x \\ y \\ z \end{bmatrix} = x - 2y + 2z$. Define T^t and find $T^t(f)$.

Q3 [12 pts]. Let $V = \mathbb{R}^3$ and its dual be V^*.

1) **[6]** Find the dual basis of $G = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

2) **[6]** Find a basis E of V whose dual basis $E^* = \{x + y + z, x - y - z, x - y + z\}$.

Q4 [12 pts]. Let $W = \{(1, -1, -2, -1), (2, -2, 3, 2), (3, -3, 1, 1), (8, -8, 5, 4)\}$

1) **[6]** Find a basis and the dimension of $\mathcal{U} = Span\ W$.

2) **[6]** Find a basis and the dimension of $\mathcal{U}^\circ$.