

MATH 555 Introduction to Commutative Algebra

Description

Basics of rings and ideals. Rings of fractions, integral dependence, valuation rings, discrete valuation rings, Dedekind domains, fractional ideals. Topologies and completions, filtrations, graded rings and graded modules. Dimension theory.

Textbook

M. F. Atiyah & I. G. Macdonald, Introduction to Commutative Algebra, Addison-Wesley, 1969. Paperback edition, Perseus Publishing, December 1994.

Syllabus

| Week | Chapter | Material |
|-------|---------|---|
| 1-2 | 1 | Rings and ideals; Prime and maximal ideals; Nilradical and Jacobson radical; Operations on ideals; Extension and contraction |
| 3-4 | 2 | Modules; Nakayama's Lemma; Exact sequences Tensor products of modules; Exactness |
| 5 | 3 | Rings and modules of fractions; Local properties Extension and contraction in rings of fractions |
| 6-7 | 5 | Integral dependence and valuations; Going-up and going-down Valuation rings; Hilbert's Nullstellensatz |
| 8 | 6 | Chain conditions Noetherian and Artinian modules |
| 9 | 7 | Noetherian rings, Hilbert's Basis Theorem Hilbert's Nullstellensatz (weak and strong versions). Noether's Normalization Lemma. |
| 10 | 8 | Artinian rings Structure Theorem |
| 11-12 | 9 | Krull dimension; Discrete valuation rings; Dedekind domains Ring of integers of an algebraic number field; Invertibility |
| 13-15 | 10 | Topologies and completions; Completion via Cauchy sequences Completion via inverse limit; Exactness of completion; I -adic topology Filtrations; Graded rings and modules; Associated graded ring |

Grading Policy

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| HW 1 (Chapters 1-3, 5) [20 problems] | 20 |
| Midterm Exam (Chapters 1-3, 5) [duration = 6 hours] | 40 |
| HW 2 (Chapters 6-10) [30 problems] | 30 |
| Final Exam (Chapters 6-10) [duration = 6 hours] | 60 |
| TOTAL | 150 |