KING FAHD UNIVERSITY OF PETROLEUM &MINERALS

Department of Mathematics

MATH 555 (Introduction to Commutative Algebra)

Semester 251 (Fall 2025)

Prof. Jawad Abuihlail

Description: Basics of rings and ideals. Rings of fractions, integral dependence, valuation rings, discrete valuation rings, Dedekind domains, fractional ideals. Topologies and completions, filtrations, graded rings and graded modules. Dimension theory.

Prerequisite: MATH551.

Textbook:

M. F. Atiyah & I. G. Macdonald, Introduction to Commutative Algebra, Addison-Wesley, 1969. Paperback edition, Perseus Publishing (1994).

Main References (Books):

(R1) R. Y. Sharp, Steps in Commutative Algebra, Cambridge University Press, 2nd edition (2010).

(R2) H. Matsumura, *Commutative Algebra*, Cambridge University Press, 2nd edition (1980).
https://aareyanmanzoor.github.io/assets/matsumura-CA.pdf

(R3) P. Grillet, *Abstract Algebra*, 2nd edition, Springer (2007).
https://link.springer.com/book/10.1007/978-0-387-71568-1

(R4) R. Wisbauer, *Foundations of Module and Ring Theory. A Handbook for Study and Research*, Gordon and Breach (Philadelphia 1991).
https://www.math.uni-duesseldorf.de/~wisbauer/book.pdf

Grading:

Midterm Exam	Assignments	Projects/Presentations	Final Exam
25%	25%	10%	40%

Exams:

	Midterm	Final
Date	20.10.2025, 7:00 PM	**TBA**

Attendance: Students are expected to attend all lecture classes.

➢ If a student misses a class, he/she is responsible for any announcement made in that class.

➢ A DN grade will be awarded to any student who accumulates more than 20% unexcused absences or 33% excused and unexcused absences

Objectives:

(1) Master the basic results in the Theory of Commutative Rings and Ideal Theory
(2) Master the basic results and techniques in the theory of Modules over Commutative Rings

Learning Outcomes:

Upon successful completion of this course, the student should be able to

Code	CLO
1	**Knowledge and Understanding**
1.1	Demonstrate an exhaustive mastery of the basics of rings and ideals, including nil and Jacobson radicals, extension and contraction of ideals.
1.2	Demonstrate an exhaustive mastery of the basics of modules over commutative rings and tensor product and exactness results, as well as apply Nakayama Lemma.
1.3	Demonstrate an exhaustive mastery of rings and modules of fractions and apply various local-global results.
2	**Skills**
2.1	Prove and apply results on integral dependence and valuation rings, including going-up and going-down theorems, and Hilbert's Nullstellensatz.
2.2	Prove and apply results on chain conditions and Noetherian and Artinian rings.
2.3	Prove and apply results on discrete valuation rings and Dedekind domains.
2.4	Prove and apply results on linear and local topologies and completions via filtrations, graded rings and modules.
3	**Values**
	Manage complex ethical and professional issues and make informed judgements on ethical codes and practices.

Detailed Syllabus

Week	Chapter	Section
1	**Categories and Functors**	Handout
2-3	**1: Rings and Ideals**	1.1 Rings and ring homomorphisms
		1.2 Ideals. Quotient rings
		1.3 Zero-divisors. Nilpotent elements. Units.
		1.4 Prime ideals and maximal ideals
		1.5 Prime ideals and maximal ideals
		1.6 Operations on ideals
		1.7 Extension and contraction
4-6	**2: Modules**	2.1 Modules and module homomorphisms
		2.2 Submodules and quotient modules
		2.3 Operations on submodules
		2.4 Direct sum and product
		2.5 Finitely generated modules
		2.6 Exact sequences.
		2.7 Tensor product of modules
		2.8 Restriction and extension of scalars
		2.9 Exactness properties of the tensor product
		2.10 Algebra
		2.11 Tensor product of algebras
7	**3: Rings and Modules of Fractions**	3.1 Local properties
		3.2 Extended and contracted ideals in rings of fractions
8-9	**5: Integral Dependence and Valuations**	5.1 Integral dependence
		5.2 The going-up theorem
		5.3 Integrally closed integral domains. The going-down theorem
		5.4 Valuation rings
10	**6: Chain Conditions**	
11	**7: Noetherian Rings**	
12	**8: Artin Rings**	
13-14	**9: Discrete Valuation Rings and Dedekind Domains**	9.1 Discrete valuation rings
		9.2 Dedekind domains
		9.3 Fractional ideals
15	**10: Completions**	10.1 Topologies and completions
		10.2 Filtrations

Projects/Presentations:

P1	Prüfer Rings and Domains
P2	Graded Rings and Modules
P3	Dimension Theory