King Fahd University of Petroleum and Minerals

MX-Program

Math557 – 231 Exam 1

Name:___________________________________ ID#_____________________ Sec#___________

Q#	Marks Obtained	Total Marks
1		10
2		10
3		10
4		17
5		10
6		10
7		15
8		10
9		8
Total		100

Q1. (10 points) Write v_4 as a linear combination of v_1, v_2, v_3.

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}, \quad v_4 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

Hint: Find c_1, c_2, c_3 by solving $c_1 v_1 + c_2 v_2 + c_3 v_3 = v_4$ and then write

$$v_4 = c_1 v_1 + c_2 v_2 + c_3 v_3$$

Q2. (10 points) Use partial pivoting to solve the linear system,

$$x_1 + x_2 + x_3 = 5$$
$$7x_1 + 5x_2 - x_3 = 8$$
$$2x_1 + x_2 + x_3 = 7$$

Answer without partial pivoting is not acceptable.

Q3. (10 points) Find LU factorization of $A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 5 & 2 & 4 & 0 \\ 0 & 3 & 3 & 6 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

Q4. (17 points) Solve the Poison problem using LU factorization of the tridiagonal coefficient matrix with $n = 3$,

$$-u'' = x + 1, \qquad x \in [0, 1], \qquad u(0) = 1, \qquad u(1) = 0.5\,.$$

Q5. (8+2 points) Consider the square matrices,

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}, O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, and\ B = \begin{bmatrix} A & A^2 \\ O & A \end{bmatrix}.$$

Use block matrices to find, a) B^{-1}. b) $\det(B)$.

Q6. (10 points) Compute A^{-1} for $A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$ using any technique.

(Not using a calculator)

Q7. (8+7 points) Compute the following:

(a) LDL^* factorization of $A = \begin{bmatrix} 3 & 4+3i \\ 4-3i & 5 \end{bmatrix}$.

(b) Cholesky factorization $LL^*$of $A = \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix}$.

Q8. (2+2+2+4 points) Use Theorem 4.3 and answer the following: Answer with justification.

(a) Which necessary condition is not satisfied for $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$ to be positive definite.

(b) Which necessary condition is not satisfied for $A = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$ to be positive definite.

(c) Can this $A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$ to be positive definite. If not, why?

(d) Show that all the necessary condition are satisfied for $A = \begin{bmatrix} 4 & 4-3i \\ 4+3i & 5 \end{bmatrix}$ to be positive definite.

Q9. (8 points) Write quadratic form of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ and show that it is positive definite.