KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS
DEPARTMENT OF MATHEMATICAL SCIENCES
SEMESTER 211/ 2021

MATH 564

# Stochastic Analysis in Finance

**Instructor: Dr. Boubaker Smii**

**BOOK:**

**T. Mikosch.** Elementary Stochastic Calculus with Finance in View. World Scientific Publishing Co. Pte. Ltd. 1998.

**S.M. Ross*.** Introduction to Probability Models, 10th Edition. Academic Press, 2010.

**Course Description:** Stochastic processes, Gaussian processes, Brownian motion, Itô stochastic integral, the Itô lemma. Introduction to stochastic differential equations, Geometric Brownian motion, financial examples. Feynman-Kac formula, Girsanov Formula and application to Black Scholes PDE and formula.

**Pre-requisite:** Graduate Standing

## COURSE OBJECTIVES

Stochastic processes and stochastic differential equations play a basic and steadily growing role in the description of phenomena occurring in the natural, technical and economical world.

The main objectives of the current course are:

* Provide the students with the basic mathematical instruments for the understanding of this important area of mathematics.

* Give them access to a very active area of contemporary mathematical research.

* Put them in a position to actively handle problems arising from real world applications.

## COURSE OUTCOMES

*Students will be able to analyse and solve some stochastic differential equations.

*They will have the basis for profitably attending future lectures related to more advanced topics and use SDE's in research, both at universities and industrial institutions.

* They will be at ease in handling problems of stochastic analysis for modeling in different application areas such as financial mathematics.

**Syllabus:**

| Week | Date | Section | |
|---|---|---|---|
| 1 | Aug.29-Sep.2 | 1 | 1.1 Sample Spaces and Events, Probabilities defined on Events.<br>1.2 Conditional Probabilities, Independent events, Bayes's Formula. |
| 2 | Sep.5-9 | 2 | 2. Random variables:<br>2.1. Discrete random variables.<br>2.2. Continuous random variables |
| 3 | Sep.12-16 | 3 | 3.1. Independence and Dependence of random variables.<br>3.2. Expectation of random variables and probability distributions |
| 4 | Sep.19-**23*** | 3 | 3.2.1. Probability distribution of continuous random variables. |
| | **September 23** | | **National day** |
| 5 | Sep. 26- 30 | 4 | 4. Stochastic processes. |
| 6 | Oct. 3-7 | 4 | 4.1. Examples of stochastic processes. |
| 7 | Oct. 10-14 | 4 | 4.1.1. Markov chains |
| 8 | Oct.17-21 | 4 | 4.2. Brownian motion: Defining properties<br>4.3. Processes derived from Brownian motion |
| 9 | Oct.24-28 | 4 | 4.4. Geometric Brownian motion.<br>4.4. Applications of Geometric Brownian motion |
| 10 | Oct.31-Nov.4 | 5 | 5.1. The Riemann-Steiltjes integral<br>5.2. Itô stochastic integral: A motivating example |
| 11 | Nov.7-11 | 5 | 5.3. Itô stochastic integral for simple processes |
| 12 | Nov. 14- 18 | 6 | 6.1. Itô formula: A simple version of the Itô lemma |
| 13 | Nov. 21-25 | 6 | 6.2. Itô lemma and applications |
| | Nov. 28-Dec.2 | ---- | Midterm Break: Nov. 28- Dec.2 |
| 14 | Dec. 5-9 | 7 | 7.1. Stochastic Differential equations (SDEs)<br>7.2. Solving SDEs. |
| 15 | Dec. 12- 16 | 7-8 | 7.3. Linear stochastic differential equations<br>8.1. Applications of Stochastic Calculus in Finance |
| | Dec. 19-20 | 8 | 8.2. The Black-Scholes Option Pricing Formula<br>8.3. A Mathematical Formulation of the Option Pricing Problem. |

**Grading policy:**

**Midterm Exam: 30%** **Quizzes(10) &Projects(20): 30%**

**Final Exam: 40%**

**Midterm Exam: November 2nd , 2021. Materials: 1.1-----4.4**