MID-TERM EXAM MATH 565
KFUPM

Problem 1: Does the problem

$$\frac{dy}{dt} = \left[2 + (t-3)^2\right]^{-1} \left[6 + (y-5)^2\right]^{-1},\ y(3) = 5$$

have a unique solution? If so, construct an appropriate rectangle.

Problem 2: Estimate the difference $x(t) - y(t)$ between two solutions of the problem

$$x' = t^2 + 2\sin(3x),\ x(0) = x_0,\ y(0) = y_0.$$

(Hint: Use Gronwall inequality)

Problem 3: Find the Hamiltonian function of $\theta'' + \sin\theta = 0$ and deduce whether the system is conservative or not.

Problem 4: Find and classify the critical points of

$$\begin{cases} x_1' = x_1 - x_2 - x_1^2 + x_1 x_2 \\ x_2' = -x_1^2 - x_2 \end{cases}$$

Problem 5: (a) Classify the critical points of

$$\begin{cases} x' = y + y^2 e^x \\ y' = x \end{cases}$$

(b) Does the system have closed trajectories?

Problem 6: Show that the system

$$\begin{cases} x' = -y + f(r)x \\ y' = x + f(r)y \end{cases}$$

where $r = \sqrt{x^2 + y^2}$, can be transformed to

$$\begin{cases} r' = r f(r) \\ \theta' = 1 \end{cases}$$

Problem 7: Use Poincare-Bendixson theorem to find a region where the problem

$$x'' + x = (1 - x^2 - 2x'^2)x'$$

has at least one periodic solution.