FINAL EXAM MATH 565
KFUPM

**Problem 1**: Consider the sytem

$$\begin{cases} x' = -x^3 \\ y' = -y(x^2 + y^2 + 1) \\ z' = -\sin z \end{cases}$$

(a) Find the equilibrium points.
(b) What can you say about the stability of the origin?
(c) Find some invariant sets
(d) Linearize around the origin and decide about the stability of the origin.
(e) Using a Lyapunov functional, is the origin asymptotically stable?

**Problem 2**: For the system

$$\begin{cases} x' = -2y + yz \\ y' = x - xz \\ z' = xy \end{cases}$$

(a) Linearize around the origin and conclude
(b) Find a Lyapunov function in the form $V(x) = C_1x^2 + C_2y^2 + C_3z^2$ and conclude.

**Problem 3**: Let

$$\begin{cases} x' = x - y - x^3 - xy^2 \\ y' = x + y - x^2y - y^3 \\ z' = \beta z, \ \beta \in R \end{cases}$$

(a) Linearize around the solution $(\cos t, \sin t, 0)^T$ and check that

$$\Phi(t) = \begin{pmatrix} e^{-2t}\cos t & -\sin t & 0 \\ e^{-2t}\sin t & \cos t & 0 \\ 0 & 0 & e^{\beta t} \end{pmatrix}$$

is the fundamental matrix.
(b) Apply Floquet theorem.

**Problem 4**: Does the system below have periodic solutions?

$$\begin{cases} x_1' = x_2 + x_2x_1^2 \\ x_2' = 2 + x_1x_2 \end{cases}$$

**Problem 5**: For the system

$$\begin{cases} x' = x + y - x^3 - xy^2 \\ y' = -x + y - x^2y - y^3 \\ z' = -z \end{cases}$$

(a) Demonstrate the existence of a limit cycle $x^2 + y^2 = 1,\ z = 0$
(b) After linearizing, what can you sat about the stability of the origin?
(c) Use cylindrical coordinates to establish the stability of the limit cycle.

**Problem 6**: Consider the system

$$\begin{cases} x' = -y - x\sqrt{x^2 + y^2} \\ y' = x - y\sqrt{x^2 + y^2} \end{cases}$$

(a) Show that the origin is a center of the linearized system, however in fact it is a spiral (using polar coordinates).
(b) Find a Lyapunov function and conclude.