King Fahd University of Petroleum and Minerals
Department of Mathematics
SYLLABUS
Semester I: 2024-2025 **(241)**

Instructor: Dr. N. Tatar
Course #: MATH 565
Title: Advanced Ordinary Differential Equations I

Textbook: Nonlinear Differential Equations and Dynamical Systems by F. Verhulst
(Second Edition, 1996. Revised 2006)

Objectives:

The course aims to reinforce students' knowledge of the concepts of existence, uniqueness, continuation, asymptotic behavior and stability of solutions to ordinary differential equations.

Course description:

Existence, uniqueness and continuity of solutions. Linear systems, solution space, linear systems with constant and periodic coefficients. Phase space, classification of critical points, Poincaré-Bendixson theory. Stability theory of linear and almost linear systems. Stability of periodic solutions. Lyapunov's direct method and applications.

Prerequisites:

MATH 435

Learning outcomes:

Upon successful completion of this course, a student should be able to:

- Solve 1^{st} order linear systems with constant coefficients.
- Prove existence, uniqueness and continuation of solutions to 1^{st} order linear and nonlinear systems.
- Analyze the asymptotic behavior of solutions to linear, almost linear and periodic systems.
- Obtain phase-portrait of 2 and 3-dimensional autonomous systems.
- Analyze periodic solutions by applying the Poincaré-Bendixson theorem.
- Prove stability of solutions to linear, almost linear and periodic systems not only by the method of linearization but also by the Lyapunov's direct method.

Week	Date	Sec	Topics	Suggested Homework Problems
1	Aug 25-29	1.2 1.3	Existence and uniqueness Gronwall's inequality	
2	Sep 1-5	2.1 2.2	Phase space, orbits Critical points and linearization	
3-4	Sep 8-19	2.3 2.4 2.5	Periodic solutions First integrals and integral manifolds Evolution of a volume element, Liouville's theorem	
5	Sep 24-26	3.1 3.2	Two-dimensional linear systems Remarks on 3-dimensional linear systems	
6	Sep 29-Oct 3	3.3	Critical points of nonlinear equations Practice session	

7	Oct 6-10	4.1 4.2	Bendixson's criterion Geometric auxiliaries, preparation for the Poincaré-Bendixson theorem	
8	Oct 13-17	4.3	The Poincaré-Bendixson theorem	
9	Oct 20-24	4.4 4.5	Applications of the Poincaré-Bendixson theorem Periodic solutions in R^n	
10	Oct 27-31	5.1 5.2	Simple examples Stability of equilibrium solutions	
11	Nov 3-7	5.3 5.4	Stability of periodic solutions Linearization	
12	Nov 17-21	6.1 6.2 6.3	Equations with constants coefficients Equations with coefficients which have a limit Equations with periodic coefficients	
13	Nov 24-28	7.1 7.2 7.3	Asymptotic stability of the trivial solution Instability of the trivial solution Stability of periodic solutions of autonomous equations	
14	Dec 1-5	8.2 8.3	Lyapunov functions Hamiltonian systems and systems with first integrals	
15	Dec 8-12	8.4	Applications and examples	
16	Dec 15-16		Practice session	

Grading:

Midterm Exam	[Secs. 1.2-4.5]	35%
Homework assignments		20%
Presentations		10%
Final Exam	[Comprehensive]	35%