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Department of Mathematics, KFUPM

Math 566 (231): Fractional Differential Equations

Instructor: Khaled Furati

Midterm Exam, Duration: 100 minutes

Student Name: ………………………………………………

Subscripts: $D_{a+} = {}_aD_x$, $D_{b-} = {}_xD_b$

1) Let $f \in C^1[0,5]$. Find ${}^{GL}D_{1+}^{1.7} f\big|_{x=3}$ if

$$f(1) = -1, \qquad f'(1) = 7, \qquad {}^{C}D_{1+}^{1.7} f|_{x=3} = 4.$$

2) Use fractional integration by parts formula to evaluate the integral

$$\int_0^1 x^{-1/2} \left[I_{1-}^{1/2} x^{1/2} \right] dx.$$

3) Provide the most general appropriate initial conditions for the equation

$${}^{RL}D_{1+}^{5/2} y - e^x y = \sin x.$$

4) Given $0 < \alpha < 1$ and $\mu > 0$, use series expansion definitions to determine the scalars (b, c) and the function $g(t)$ such that

$${}^{C}D_{0+}^{\alpha} E_\mu(\lambda t^\mu) = t^{-\alpha} E_{b,c}(\lambda t^\mu) + g(t), \qquad t > 0.$$

5) Let $y(x) = x^2$ on $[0, 3]$ and zero otherwise. Find

$$\left.\frac{d^{5/4} y}{d|x|^{5/4}}\right|_{x=2}.$$

6) Derive the VIE satisfied by the solution $y \in C^2[0,4]$ of the Cauchy problem

$${}^{c}D_{0+}^{7/5} y + 4\ {}^{c}D_{0+}^{3/5} y = 0, \qquad y(0) = 2, \qquad y'(0) = 1.$$

7) True or false. Justify you answer.

a) $f(x) = \sin x \quad \Longrightarrow \quad D^3\left[{}^{RL}D_{0+}^{1/2} f\right] = {}^{RL}D_{0+}^{7/2} f.$

b) $f(x) = \dfrac{\cos x}{\sqrt{x}} \quad \Longrightarrow \quad f \in C_{1/3}[0, \pi].$

c) $f \in L^1(0,1) \quad \Longrightarrow \quad (I_{0+}^{\alpha} f)(0) = 0 \quad$ for all $\alpha > 1$.

Formula

$$I_{0+}^{\alpha} D_{0+}^{\alpha} f(t) = f(t) - \sum_{k=1}^{n} \frac{D_{0+}^{\alpha-k} f(0)}{\Gamma(\alpha - k + 1)}\, t^{\alpha-k}, \qquad {}^{c}D_{a+}^{\alpha} f \coloneqq D_{a+}^{\alpha}\left[f - \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{k!}\,(x-a)^k\right],$$

$$\frac{d^\alpha y}{d|x|^\alpha} = \frac{1}{2\cos\frac{\alpha\pi}{2}}\,[D_{a+}^{\alpha} y + D_{b-}^{\alpha} y], \qquad \int_a^b \phi(x)\, I_{a+}^{\alpha}\psi(x)\, dx = \int_a^b \psi(x)\, I_{b-}^{\alpha}\phi(x)\, dx,$$

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}.$$

Q	1	2	3	4	5	6	7	Total
Max	7	7	7	7	7	7	9	51
Points								