Show your work

King Fahd University of Petroleum & Minerals
Department of Mathematics
Math 571: Numerical analysis of ODEs
Final Exam (232)
Dr. Khaled Furati

Student Name: …………………………………………… **Duration: 150 min**

Q	1	2	3	4	5	6	7	8	Total
Max	10	10	10	10	10	10	10	10	80
Points									

Q1 Consider the method

$$y_{n+1} = y_{n-1} + 2h\, f(x_n, y_n), \qquad n \geq 1.$$

a) Show that the method is a special case of the general multistep method formula.
b) Obtain the characteristic polynomials ρ and σ for this method and use them to confirm that the method is consistent.

Q2 Construct the second order consistent 2-step method of the form

$$y_{n+1} = -4y_n + ay_{n-1} + h\,(bf_{n+1} + 2f_n + cf_{n-1}), \qquad n \geq 1.$$

Q3 If shooting method is to be used to approximate the solution of the following BVP, formulate the **nonlinear equation** for $y'(0) = s$ that needs to be solved using Newton's method.

$$y'' + y^2 = 0, \qquad y(0) = 1, \qquad y(1) = 2.$$

Q4 For which values of c the following method is zero-stable?

$$y_{n+2} - cy_{n+1} = \frac{h}{2}(3f_{n+1} - f_n).$$

Q5 Use Schur's criterion to determine the interval of absolute stability of the method

$$y_{n+2} - y_{n+1} = \frac{h}{2}[3f_{n+1} - f_n].$$

Q6 Consider the BDF method

$$y_{n+2} - \frac{4}{3}y_{n+1} + \frac{1}{3}y_n = \frac{2}{3}hf_{n+2}\,.$$

a) Use the characteristic polynomials to show that the method is consistent.
b) Use the root condition to show that the method is zero-stable.
c) Is the method convergent? Justify.

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Show your work

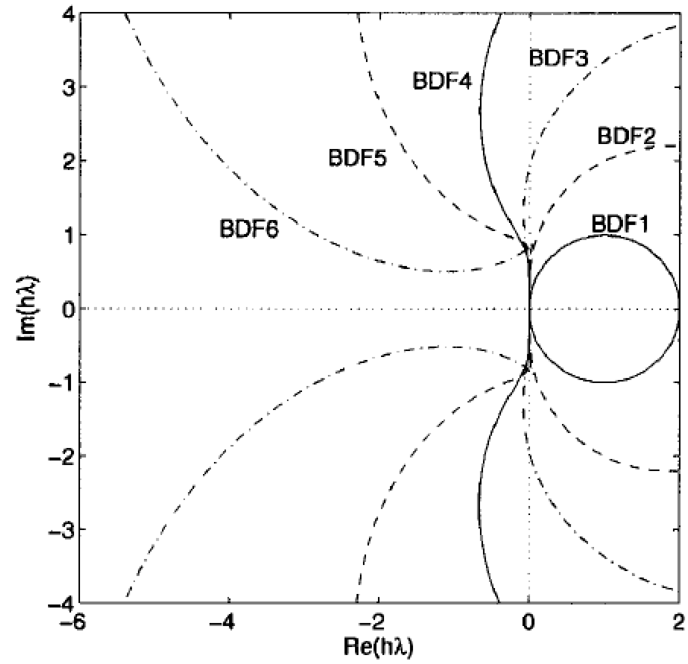
Q7 Consider the following two-point BVP:

$$y'' + e^x y = x, \qquad 0 < x < 1,$$
$$y(0) = 2, \qquad y(1) = 0.$$

a) Develop a second-order finite difference scheme with N intervals of length h.
b) Show that the truncation error is of order 2.
c) Write the scheme in **matrix-vector formulation** when $N = 4$.

Q8 Consider the stability regions of BDF methods. Which of the following is true? Justify.

a) BDF1 is A-stable
b) BDF6 is $A(\alpha)$-stable.



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Hint.
Theorem Let

$$y_{n+1} = \sum_{j=0}^{p} a_j\, y_{n-j} + h \sum_{j=-1}^{p} b_j\, f(x_{n-j}, y_{n-j}), \qquad n \geq p, \qquad |a_p| + |b_p| \neq 0.$$

Then,

$$\text{The method is consistent} \iff \sum_{j=0}^{p} a_j = 1, \qquad -\sum_{j=0}^{p} j a_j + \sum_{j=-1}^{p} b_j = 1. \quad (C)$$

$$\tau(h) = O(h^m) \text{ for all } y \in C^{m+1} \iff (C) \text{ holds } \& \sum_{j=0}^{p} (-j)^i a_j + i \sum_{j=-1}^{p} (-j)^{i-1}\, b_j = 1.$$