Show your work

King Fahd University of Petroleum & Minerals
Department of Mathematics
Math 571: Numerical analysis of ODEs
Midterm Exam (232)
Dr. Khaled Furati

Student Name: …… **Duration: 120 min**

Q	1	2	3	4	5	Total
Max	10	10	10	10	10	50
Points						

Note

- $f = f\big(x, y(x)\big)$ is smooth and satisfies Lipschitz condition in y.
- $f_n \coloneqq f(x_n, y_n)$ and y_n is the numerical approximation of $y(x_n)$.
- Truncation error for one step methods, $T_n \coloneqq \frac{1}{h}[y(x_{n+1}) - y(x_n)] - \Phi(x_n, y(x_n); h)$.

Problem 1 Consider the IVP

$$y'(x) = \sin y + e^x, \qquad x \in [1, 2],$$
$$y(1) = 0.$$

Define the **explicit Euler** method and show that the global error is of order $O(h)$.

Problem 2 Consider the method

$$y_{n+1} = y_n + \frac{h}{4}[f_{n+1} + 3f_n].$$

a) Calculate the truncation error.
b) Derive the stability function $R(z)$.
c) Determine the interval of absolute stability.

Problem 3 Consider the three-stage RK method given by the following tableau.

a) Write the RK method.
b) Show that the method is of **second order**.

0	0		
1	1	0	
1	1/3	2/3	0
	1/2	0	1/2

See next page ------->

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Problem 4 The outcome of a numerical experiment is shown in the table. Calculate the order of the method.

h	x_n	$e_n = y(x_n) - y_n$
0.04	1.0	-1.70E-02
	2.0	-1.83E-02
	3.0	-2.78E-03
	4.0	1.53E-02
	5.0	1.94E-02
0.02	1.0	-8.46E-03
	2.0	-9.13E-03
	3.0	-1.40E-03
	4.0	7.62E-03
	5.0	9.63E-03

Problem 5 Consider the following questions. **For T/F questions, provide a brief justification.**

a) (T/F) The Butcher tableau for the method $y_{n+1} = y_n + \frac{1}{2}h[\, f_n + f(x_{n+1}, y_n + hf_n)\,]$ is

1		1
1	1	0
	1/2	1/2

b) (T/F) Consistency of a numerical method means that the **method** will converge to the differential equation as the step size goes to 0.

c) (T/F) Runge-Kutta methods are always explicit.

d) **Define** the order of a numerical method.

e) Describe how to find the order of a method **numerically**.

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