**King Fahd University of Petroleum & Minerals**
**Department of Mathematics and Statistics**
**Math 572: Numerical Methods for Partial Differential Equations**
**Midterm Exam , Fall Semester 211**

**Problem 1:**

(a) Consider the variational problem: find $u \in V$ such that $a(u, v) = L(v)$ for all $v \in V$, where

$$a(u, v) = \int_0^1 u'v'dx + \int_0^{1/4} uvdx, \qquad L(v) = \int_0^1 2x^2 v(x)dx,$$

and $V = H^1(0, 1)$. Prove the bilinear form is coercive in $V$ equipped with the norm $\|u\|_V^2 = \int_0^1 (u'^2 + u^2)dx$;

(b) Is the bilinear form $a(u, v) = \int_0^1 (u'v' - 20uv)dx$ coercive in $H_0^1(0, 1)$. Justify your answer.

**Problem 2:**

(a) Give the variational formulation of the following boundary value problem:

$$-u'' + 3u = 4, \quad \text{for } x \in (0, 2), \;\; u'(0) - 2u(0) = 0, \;\; u(2) = 0.$$

(b) Assemble the Ritz-Galerkin system for this problem when using **two** linear finite elements.

**Problem 3:**

Let $V_h$ be the finite dimensional space of continuous piecewise **quadratic** functions over the partition of the interval $(0, 1)$ into $n$ sub-intervals of size $h = 1/n$. The functions in $V_h$ over each sub-interval are determined by their value at the end points and at the midpoint. Let $u$ be the solution of the elliptic operator

$$\mathcal{L}u(x) = f(x), \quad x \in (0, 1), \;\; u(0) = u(1) = 0,$$

and $u_h$ be the Galerkin finite element approximation (that is the Galerkin FE solution) and $u_I$ the interpolant of $u$ in $V_h$.

(a) Show that

$$\|u' - u_I'\| \leq Ch^2 \|u'''\|,$$

where $h = 1/n$ and $\|v\|^2 = \int_0^1 u^2 dx$.

(b) prove the estimate

$$\|u - u_h\| \leq Ch^3 \|u'''\|.$$

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Good luck

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