King Fahd University of Petroleum and Minerals
Department of Mathematics

MATH 578: Applied Numerical Methods II

Midterm Exam: Semester 231 (120 minutes)

– –

Note: Use of electronic devices such as smartphones are not allowed. Each question has five points.

Total Points: 25.

Instructror: Abdullah Shah (*abdullah.shah.1@kfupm.edu*.sa)

NAME: STUDENT ID:

Q1. Use finite difference method with $h = k = 1$ to write the matrix form of

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad 0 < x < 3, \quad 0 < y < 3,$$

subject to the boundary conditions

$$\begin{aligned} u(x,0) &= 0, \qquad u(x,3) = x \\ u(0,y) &= 0, \qquad u(3,y) = y. \end{aligned}$$

NAME: STUDENT ID:

Q2. Approximate the solution to the equation

$$\begin{aligned}
\frac{\partial^2 u}{\partial t^2} &= \frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \quad 0 < t; \\
u(0,t) &= u(1,t) = 0, \qquad\qquad\qquad 0 < t, \\
u(x,0) &= \sin 2\pi x, \qquad 0 \le x \le 1, \\
\frac{\partial u(x,0)}{\partial t} &= 2\pi \sin 2\pi x, \quad 0 \le x \le 1,
\end{aligned}$$

by finite difference method with $h = 0.5$ and $k = 0.1$. Find the solution $u(x,t)$ at $t = 0.2$.

NAME: STUDENT ID:

Q3. Use $h = k = 0.25$, write the matrix (discrete) form $Aw^j = w^{j-1}$ of

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial^2 x} = 0, \qquad 0 < x < 1, \quad 0 < t;$$

subject to the boundary and initial conditions

$$\begin{aligned} u(0,t) &= u(1,t) = 0, \qquad 0 < t; \\ u(x,0) &= x(1-x), \end{aligned}$$

using backward difference method.

NAME: STUDENT ID:

Q4. What are the key components of Finite Element Method. Write the weak form (varitional form) of the following boundary value problem.

$$-u''(x) = f(x) \qquad 0 < x < 1, \qquad u(0) = u(1) = 0.$$

NAME: STUDENT ID:

Q5. Define of the following spaces.

i. $C(\Omega)$ ii. $C^{\infty}(\Omega)$ iii. $L^2(\Omega)$ iv. $H^1(\Omega)$ v. $H_0^2(\Omega)$, where $\Omega = [0\ 1]$.