King Fahd University of Petroleum and Minerals
Department of Mathematics

MATH 578: Applied Numerical Methods II

Midterm Exam-II: Semester 241 (120 minutes)

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Note: Use of electronic devices such as smartphones are not allowed.

NAME: :...................................... STUDENT ID:......................................

Total Points: 25. Each question has five points.

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Q1. Consider the equation

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0.$$

(a) Complete the table below by (i) identifying the numerical schemes, (ii) specifying whether they are implicit or explicit, and (iii) describing their stencil configuration.

S.No.	Difference Equations (Numerical Scheme)	Name	Implict/Explicit	Stencil
(1).	$U_j^{n+1} = U_j^n - \frac{ak}{2h}\left(3U_j^n - 4U_{j-1}^n + U_{j-2}^n\right) +$			
	$\frac{a^2k^2}{2h^2}\left(U_j^n - 2U_{j-1}^n + U_{j-2}^n\right).$			
(2).	$U_j^{n+1} = \frac{1}{2}\left(U_{j-1}^n + U_{j+1}^n\right) - \frac{ak}{2h}\left(U_{j+1}^n - U_{j-1}^n\right).$			
(3).	$U_j^{n+1} = U_j^n - \frac{ak}{h}\left(U_{j+1}^n - U_j^n\right).$			
(4).	$U_j^{n+1} = U_j^n - \frac{ak}{2h}\left(U_{j+1}^{n+1} - U_{j-1}^{n+1}\right) +$			
	$\frac{a^2k^2}{2h^2}\left(U_{j+1}^n - 2U_j^n + U_{j-1}^n\right).$			

(b) Using the scheme provided in S.No. (3) with $h = 0.25$, and $k = 0.2$, solve the given equation for time $T = 0.4$ subject to the given initial and boundary conditions.

$$\begin{aligned} \frac{\partial u}{\partial t} - \frac{\partial u}{\partial x} &= 0, \quad 0 < x < 1, \qquad t > 0. \\ u(x,0) &= \begin{cases} 1 & if \quad 0.25 < x < 0.75 \\ 0 & \qquad \text{otherwise} \end{cases}, \\ u(0,t) &= 0 = u(1,t). \end{aligned}$$

Q2. Explain the 2D finite element method and discuss its advantages over the finite difference method.

Q3. Describe the different types of boundary conditions commonly used in numerical methods for solving partial differential equations.

Q4. Define Riemann problem. Consider the Riemann problem for the equation

$$u_t + f(u)_x = 0,$$

with $f(u) = \frac{1}{2}u^2$ and initial condition

$$u(x,0) = \begin{cases} u_l & \text{if} \quad x < 0 \\ u_r & \text{if} \quad x > 0 \end{cases},$$

Find the solution when (i) $u_l = 0, u_r = 1$ and (ii) $u_l = 1, u_r = 0$. (iii) Also sketch your solutions $u(x,t)$ at $t = 0$ and $t = 2$.

Q5. Consider the partial differential equation

$$u_t + 3u_x + u = 0,$$

with the initial condition

$$u(x, 0) = \sin(x).$$

1. Solve the equation using the method of characteristics.
2. Verify that your solution satisfies both the partial differential equation and the initial condition.