Department of Mathematics, King Fahd University of Petroleum & Minerals, Math 581 Final Exam, 2024-2025 (242)
Time Allowed: 160 Minutes

Name: ID#:

- Mobiles, calculators, and smart devices are not allowed in this exam.
- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. **No points for answers without justification.**

Question #	Marks	Maximum Marks
1		12
2		14
3		14
4		16
5		18
6		16
Total		90

Q1: A computer centre has five expert programmers and needs to develop five application programs. The head of the computer centre, estimates the computer time (in minutes) required by the respective experts to develop the application programs as follows:

Programs	**Programmers**				
	I	**II**	**III**	**IV**	**V**
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

Find the assignment pattern that minimizes the time required to develop the application programs.

$Q2$: Consider the following linear fractional programming problem:

$$\text{(LFP) Minimize } \frac{x_1 + 3x_2 + 3}{2x_1 + x_2 + 6}$$

subject to

$$2x_1 + x_2 \leq 12$$

$$-x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$

(a) Convert the above problem into a linear programming problem.
(b) Write the Karush-Kuhn Tucker conditions for LPP in Part (a) (**Do not solve).**
(c) Is (LFP) a convex programming problem? Justify your response.

$Q3$: Solve the following Integer programming problem by Branch and Bound method

Maximize z $= 5x_1 + 4x_2$

subject to

$$x_1 + x_2 \quad \leq 5$$

$$10x_1 + 6x_2 \quad \leq 45$$

$$x_1, x_2 \geq 0 \text{ and integers.}$$

$Q4$: Consider the following linear programming problem:

Maximize $z = 5x_1 + 3x_2$

subject to

$$3x_1 + 5x_2 \leq 15$$

$$5x_1 + 2x_2 \leq 10$$

$$x_1, x_2 \geq 0.$$

The optimal table of the problem is given below:

	x_1	x_2	s_1	s_2	RHS
x_2	0	1	$\frac{5}{19}$	$-\frac{3}{19}$	$\frac{45}{19}$
x_1	1	0	$-\frac{2}{19}$	$\frac{5}{19}$	$\frac{20}{19}$
$z_j - c_j$	0	0	$\frac{5}{19}$	$\frac{16}{19}$	

(a) Use sensitivity analysis to find the variation in C_1 and b_1
(b) Let a new variable $x_3 \geq 0$ be introduced with cost 4 assigned to it in the objective function. Suppose the coefficients x_3 in the two constraints are 3 and 4, respectively. Discuss the impact of a new variable on the optimality of the given problem.
(c) Discuss the effect on the optimal solution by adding a new constraint $x_1 + x_2 \leq 3$.

$Q5(a)$: Define a convex function on a convex set. Let $f(x) = x^T Dx, x \in R^n, (D)_{n\times n}$. If D is a positive semidefinite matrix, then show that $f(x)$ is a convex function.

$Q5(b)$: Convert the following matrix form of the Karush-Kuhn Tucker system to minimize the objective function, in a convex programming problem, subject to conditions with restrictions on decision variables x_1, x_2. $\lambda_1, \lambda_2, \mu_1, \mu_2$ are Lagrangian multipliers, and s_1, s_2 represent the slack variables. All variables are non-negative and $\lambda_i s_i = 0 = \mu_1 x_i, i = 1, 2$.

$$\begin{pmatrix} 6 & -2 & -1 & 3 & -1 & 0 & 0 & 0 \\ -2 & 4 & -1 & 4 & 0 & -1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \lambda_1 \\ \lambda_2 \\ \mu_1 \\ \mu_2 \\ s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ -1 \\ 12 \end{pmatrix}$$

$Q6$: (a) Write all the steps of Karmarkar's algorithm.
(b) Consider the following LPP;
Miniimize z $= -6x_1 + 6x_2 - x_3$

subject to

$$x_1 - x_2 = 0$$

$$x_1 + x_2 + x_3 = 1$$

$$x_1, x_2, x_3 \geq 0.$$

(i) Show that LPP meets all the assumptions required to apply Karmarkar's algorithm.
(ii) Starting with $x^{(0)} = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})^T$, compute projection matrix $P = [I - B^T(BB^T)^{-1}B]$.