MATH 581 Midterm Exam (Term 242)

Department of Mathematics, King Fahd University of Petroleum & Minerals, Math 581 Midterm, 2024-2025 (242)

Time Allowed: 120 Minutes

Name: ID#:

- Mobiles, calculators and smart devices are not allowed in this exam.
- Write neatly and eligibly. You may lose points for messy work.
- Show all your work. **No points for answers without justification.**

Question #	Marks	Maximum Marks
1		12
2		12
3		12
4		10
5		12
6		12
Total		70

$Q1$: Solve by Simplex Method

Maximize z $= 2x_1 - x_2 + x_3$

subject to

$$2x_1 + x_2 \quad \leq 10$$

$$x_1 + 2x_2 - 2x_3 \quad \leq 20$$

$$x_1 + 2x_3 \quad \leq 5$$

$$x_1, x_2, x_3 \geq 0.$$

$Q2$: Solve by Dual Simplex Method

Maximize z $= -2x_1 - x_2$

subject to

$$-3x_1 - x_2 \leq -3$$

$$-4x_1 - 3x_2 \leq -6$$

$$-x_1 - 2x_2 \leq -3$$

$$x_1, x_2 \geq 0.$$

Write the solution of the primal problem from the optimal table of dual simplex.

$Q3(a)$: State and prove strong duality theorem.

(b): Use the information of the following primal problem (P) to solve the dual problem by Complementary Slackness conditions.

Maximize $z = 2x_1 + 4x_2 + 3x_3 + x_4$

subject to

$$3x_1 + x_2 + x_3 + 4x_4 \leq 12$$

$$x_1 - 3x_2 + 2x_3 + 3x_4 \leq 7$$

$$2x_1 + x_2 + 3x_3 - x_4 \leq 10$$

$$x_1, x_2, x_3, x_4 \geq 0,$$

where the solution of the primal problem is

$$(x_1, x_2, x_3, x_4) = (0, 10.4, 0, 0.4)$$

$Q4$: Consider the linear programming problem $\{Min\ c^T x : Ax = b, x \geq 0\}$, where

$$A = \begin{bmatrix} 1 & 1 & -2 & 2 \\ 1 & -1 & 2 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \ and \quad c = \begin{bmatrix} 2, & 0, & -1, & 2 \end{bmatrix}^T.$$

Apply the Revised Simplex method to solve the above problem starting with the basis $B = \{1, 2\}$.

$Q5$: Consider the following Transportation Model

	D_1	D_2	D-3	Supply
S_1	2	2	3	10
S_2	4	1	2	15
S_3	1	3	1	40
Demand	20	15	30	

(a) Find the initial basic feasible solution by North-West corner method.
(b) Find the optimal solution by u-v method.

$Q6(a)$: Write the dual of the following linear programming problem

$$Minimize \quad z = 5x_1 + 6x_2$$

subject to

$$x_1 + 2x_2 = 5$$

$$-x_1 + 5x_2 \geq 3$$

$$4x_1 + 7x_2 \leq 8$$

$$x_2 \geq 0. \ , \ x_1 \text{ is unrestricted.}$$

(b) Consider the system of equations

$$3x_1 + x_2 + 2x_3 - x_4 = a$$

$$x_1 + x_2 + x_3 - 2x_4 = 3$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

If $x_1 = 1, x_2 = b, x_3 = 0, x_4 = c$ is a basic feasible solution of the above system (where a, b and c are real constants), then $a + b + c =$ ..... (Justify your answer).