# King Fahd University of Petroleum and Minerals
# Mathematics Department

## MATH 583: Computer Graphics: Modeling and Processing

## Midterm Exam: Semester 231 (120 minutes)

**NAME:** **STUDENT ID:**

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**Problem 1:** (7 points) Consider the parametric curve

$$x(t) = -6t^3 + 9t^2, \quad y(t) = -3t^2 + 3t, \quad t \in \mathbb{R}.$$

*a)* Determine the points on the parametric curve for which the tangent is horizontal.

*b)* Determine the parameters $t$ for which the slope of the tangent is equal $-\frac{1}{3}$.

*c)* Using the blossom, compute the Bézier control points of the parametric curve over the interval $[0,1]$.

*d)* Sketch the Bézier curve approximately: Draw your sketch to scale as best you can and, when drawing the Bézier curve, show clearly the initial and final points and the initial and final slopes.

*e)* Using de Casteljau algorithm, compute the point on the parametric curve corresponding to the parameter to the parameter $t = \frac{2}{3}$.

*f)* Compute the curvature of the parametric curve at the point with parameter $t = 0$.

**Problem 2:** (4 points) A particle is animated so that it start at $(0,0)^T$ at time $t = 0$ and end at $(2,0)^T$ at time $t = 1$. The initial velocity at time $t = 0$ is equal twice its final velocity at time $t = 1$ and such that the sum of the velocities at $t = 0$ and $t = 1$ is equal to $(3,5)^T$. The motion is modeled with a degree 3 Bézier curve $P(t)$ over the interval $[0,1]$.

Give the control points of the Bézier curve.

**Problem 3:** (3 points) Let $P$ be a polynomial of degree 2 with control points $(p_0, p_1, p_2)$ over the interval $[0,1]$ with

$$p_0 = 1, \ p_1 = 3, \ p_2 = 0.$$

Denotes by $p$ the blossom of $P$

*a)* Compute the Bézier control points of $P$ over the interval $[\frac{1}{2}, 1]$.

*b)* Compute the value of $p\left(\frac{1}{2}, \frac{3}{2}\right)$.

**Problem 4:** (6 points) Consider the function $S(t)$ defined by

$$S(t) = \begin{cases} t^3 + 6t - 2 & \text{if } t \in [0,1] \\ 2t^3 + at^2 + bt - 3 & \text{if } t \in [1,2] \end{cases}$$

where $a$ and $b$ are real numbers.

*a)* Determine the real numbers $a$ and $b$ so that $S(t)$ is a cubic spline over the interval $[0,2]$.

*b)* Compute the de Boor control points of the obtained spline $S(t)$ in the clamped case.

*c)* Using de Boor algorithm, compute the value $S\left(\frac{3}{2}\right)$.

*d)* Deduce the de Boor control polygon of the spline $S(t)$ over the interval $[0, \frac{3}{2}]$.