KING FAHD UNIVERSITY OF PETROLEUM &MINERALS

Department of Mathematics

MATH 655 (Applied & Computational Algebra)

Graphs of Rings and Semirings

Semester 242 (Spring 2025)

Prof. Jawad Abuihlail

Description: Contents vary. Concepts and methods in algebra which have wide applications in mathematics as well as in computer science, systems theory, information theory, physical sciences, and other areas. Topics may be chosen from fields of advanced matrix theory; algebraic coding theory; group theory; Gröbner bases; or other topics of computational and applied algebra.

Prerequisite: Graduate Standing.

Textbook:

D. F. Anderson , T. Asir , A. Badawi and T. T. Chelvam, *Graphs from Rings*, Springer (2021).

https://link.springer.com/book/10.1007/978-3-030-88410-9

Main References (Books):

(R1) A. Bondy and U. S. R. Murty, *Graph Theory*, Springer (2008).
https://link.springer.com/book/9781846289699

(R2) M. Gondran and M. Minoux, *Graphs, Dioids and Semirings: New Models and Algorithms*, Springer (2008).
https://link.springer.com/book/10.1007/978-0-387-75450-5

(R3) J. Golan, *Semirings and their Applications*, Springer (2013).
https://link-springer-com.kfupm.idm.oclc.org/book/10.1007/978-94-015-9333-5

(R4) P. Grillet, *Abstract Algebra*, 2nd edition, Springer (2007).
https://link.springer.com/book/10.1007/978-0-387-71568-1

Main References (Papers):

J1: I. Beck, *Coloring of commutative ring*, J. Algebra, 116 (1988), 208-226.
J2: D. F. Anderson, P.S. Livingston, *The zero–divisor graph of a commutative ring*, J. Algebra, 217 (1999), 434-447.
J3: S. E. Atani, *The zero-divisor graph with respect to ideals of a commutative semiring*, Glas. Mat. 43(63) (2008), 309–320.

Grading:

Midterm Exam	Assignments	Projects/Presentations	Poster	Final Exam
20%	10%	24%	16%	30%

Assignments using MATLAB/Python and software related to Algebra and Graph Theory.

Exams:

	Midterm	Final
Date	25.2.2025	**TBA**

Attendance: Students are expected to attend all lecture classes.

➢ If a student misses a class, he/she is responsible for any announcement made in that class.

➢ A DN grade will be awarded to any student who accumulates more than 20% unexcused absences or 33% excused and unexcused absences

Objectives:

(1) To realize the properties of the zero-divisor graphs of rings and semirings.
(2) To understand the interplay between the algebraic properties of the rings (semirings) under consideration and the graph theoretic properties of their zero-divisor graphs.

Learning Outcomes:

Upon successful completion of this course, the student should be able to

Code	CLO
1	**Knowledge and Understanding**
1.1	Discuss **basic properties of the zero-divisor graphs** of rings and semirings.
1.2	Discuss the **colorings** of zero-divisor graphs of rings and semirings.
2	**Skills**
2.1	To compute the zero-divisor graphs of several classes of rings and semirings.
2.2	To compute the **diameter, girth and radius** of zero-divisor graphs of several classes of rings and semirings.
2.2	To compute the **genus** of zero-divisor graphs of several classes of rings and semirings.
2.3	To compute the **clique number** of zero-divisor graphs of several classes of rings and semirings.
3	**Values**
	Manage complex ethical and professional issues and make informed judgements on ethical codes and practices.

Detailed Syllabus

Week	Chapter	Section
1	**1: Introduction**	1.1 Basics in Algebra
		1.2 Basics in Graph Theory
2	**2: Distances in Zero-divisor Graphs**	2.1 Zero-divisor Graphs
		2.2 Basic Properties of Zero-divisor Graphs
3		2.3 Girth of Zero-divisor Graphs
		2.4 Diameter of Zero-divisor Graphs
4		2.5 Radius and Center of Zero-divisor Graphs
		2.6 Rings without Identity
5	**3: Properties of Zero-divisor Graphs**	3.1 Coloring of Zero-divisor Graphs
		3.2 Connectivity of Zero-divisor Graphs
6		3.3 Isomorphisms and Automorphisms
		3.4 Bipartite Zero-divisor Graphs
7		3.5 Zero-divisor Graphs of Gaussian Integers
		3.6 Cycles in Zero-divisor Graphs
February 23, 2025: Saudi Founding Day		
February 25, 2025: Midterm Exam		
8	**4: Genus of Zero-divisor Graphs**	4.1 Genus of Graphs
		4.2 Planar Zero-divisor Graphs
9		4.3 Zero-divisor Graphs with Genus One
		4.4 Zero-divisor Graphs with Genus Two
10	**5: Generalized Zero-divisor Graphs**	5.1 Ideal-based Zero-divisor Graphs
		5.2 Properties of $\Gamma_I(R)$
11		5.4 Primal Ideal-based Zero-divisor Graphs
		5.5 Annihilating-ideal Graphs
12	**6: Zero-divisor Graph Generalizations**	6.1 Noncommutative Rings
		6.2 Commutative Semigroups
13-15		6.3 Commutative Semirings

Projects/Presentations:

	I	II	Due
P1	Zero Divisor Graphs of Modules	Generalized Total Graphs	
P2	Total Graphs of Modules	Total Graphs of Semirings	
P3	Additional Graphs of Commutative Rings	Annihilator Graphs	
P4	Additional Graphs of Noncommutative Rings	Graphs of Ideals	