King Fahd University of Petroleum and Minerals
Department of Mathematics

Final Exam MATH 667-251

Problem 1: Discuss the shock/rarefaction for $z_t + [z(1-z)]_x = 0$
(a) $z_L = 0.2$ and $z_R = 0.8$
(b) $z_L = 0.8$ and $z_R = 0.2$

Problem 2: Let u be the solution of $u_t + (u^2/2)_x = 0$. Classify the waves, give the speeds and $u(x,t)$ when
(a) $u_L = 3$ and $u_R = 1$
(b) $u_L = 2$ and $u_R = -1$

Problem 3: For the Schrodinger equation $iu_t + u_{xx} = 0$, $u(0,x) = u_0(x)$
(a) Find $\hat{u}(t,\xi)$
(b) Find $u(t,x)$
(c) Show that $\|u\|_{H^s} = \|u_0\|_{H^s}$ where $\|u\|_{H^s}^2 := \int_{R^n} \left(1+|\xi|^2\right)^s |\hat{u}(t,\xi)|^2 \, d\xi$
(d) Conclude that the Cauchy problem is globally well-posed in $H^s(R^n)$, $\forall s \in R^+$.

Problem 4: Derive the fundamental solution for

$$u_t + u_{xxx} = 0.$$

Problem 5: Find the first blow up time of the spatial derivative for the solution of

$$u_t + \left(\frac{u^3}{3}\right)_x = 0, \; u_0 = A\sin x, \; A > 0.$$