KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
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DEPARTMENT OF MATHEMATICS

STAT 214: STATISTICAL METHODS FOR ACTUARIES
Term 231, Major Exam 1I, Monday November 6, 2023, 7:00PM-9:00PM

Name: ID #:

Part 1: (20points)

1. A statistical analysis of 1,000 long-distance telephone calls made from the headquarters of the Bricks and Clicks Computer Corporation indicates that the length of these calls is normally distributed, with $\mu = 240$ seconds and $\sigma = 40$ seconds, 1% of all calls will last less than how many seconds?

   a. 147.2
   b. 332.8
   c. 136.97
   d. 343.03
   e. 167.8

2. The time between unplanned shutdowns of a power plant has an exponential distribution with a mean of 20 days. Find the probability that the time between two unplanned shutdowns is less than 14 days.
   a. 0.4966
   b. 1
   c. 0.5034
   d. 0.9654
   e. 0.0346

3. The amount of time a bank teller spends with each customer has a population mean, μ= 3.10 minutes and a standard deviation, $\sigma = 0.40$ minute. If you select a random sample of 16 customers, what is the probability that the mean time spent per customer is at least 3 minutes?
   a. 0.8413
   b. 0.4013
   c. Almost zero
   d. 0.1587
   e. 0.5987

4. Sales prices of baseball cards from the 1960s are known to possess a right skewed distribution with a mean sale price of $5.25 and a standard deviation of $2.80. Suppose a random sample of 100 cards from the 1960s is selected. Describe the sampling distribution for the sample mean sale price of the selected cards.
   a. Right skewed with a mean of $5.25 and a standard error of $2.80
   b. Normal with a mean of $5.25 and a standard error of $0.28
   c. Right skewed with a mean of $5.25 and a standard error of $0.28
   d. Normal with a mean of $5.25 and a standard error of $2.80
   e. None of the above

5. The following data represent the responses (Y for yes and N for no) from a sample of 40 college students to the question "Do you currently own shares in any stocks?"

   N N Y N N Y N Y N Y N N Y N Y Y N N N Y
   N Y N N N N Y N N Y Y N N N Y N N Y N N

   If the population proportion is 0.30, determine the standard error of the proportion.
   a. 0.0725
   b. 0.0053
   c. 0.2354
   d. 0.35
   e. 0.0754

6. The head librarian at the Library of Congress has asked her assistant for an interval estimate of the mean number of books checked out each day. The assistant provides the following interval estimate: from 740 to 920 books per day. If the head librarian knows that the population standard deviation is 150 books checked out per day, and she asked her assistant for a 95% confidence interval, approximately how large a sample did her assistant use to determine the interval estimate?
   a. 125
   b. 13
   c. 11
   d. 4
   e. 22

7. The data below is the overall miles per gallon (MPG) of 2010 small SUVs. Source: Data extracted from "Vehicle Ratings," Consumer Reports, April 2010.

   24 21 23 24 34 34 34 20 20 22 22 44 32 20 20 22 20 39 20

   If the 95% confidence interval estimate for the population mean MPG of 2010 small SUVs, is (22.41205 29.69321). Which of the following is true if a 90% confidence interval for μ is constructed?
   a. It is wider than the 95% confidence interval
   b. It is narrower than the 95% confidence interval
   c. It is the same as the 95% confidence interval
   d. It is the same as the 99% confidence interval
   e. There is not enough information to determine the answer

8. In a survey of 1,200 social media users, 76% said it is okay to friend co-workers, but 56% said it is not okay to friend your boss. (Data extracted from “Facebook Etiquette at Work,” USA Today, March 24, 2010, p. 1B.). What is the width of the 95% confidence interval to estimate for the population proportion of social media users who would say it is okay to friend co-workers.
   a. 0.0483
   b. 0.4830
   c. 0.0242
   d. 0.8083
   e. 0.7117

9. Suppose we wish to test H_0: $\mu \leq 47$ versus H_1: $\mu > 47$. What will result if we conclude that the mean is greater than 47 when its true value is really 52?
   a. We have made a Type I error.
   b. We have made a Type II error.
   c. We have made a correct decision
   d. None of the above are correct.
   e. The power of the test is high

10. An appliance manufacturer claims to have developed a compact microwave oven that consumes a mean of no more than 250 W. From previous studies, it is believed that power consumption for microwave ovens is normally distributed with a population standard deviation of 15 W. A consumer group has decided to try to discover if the claim appears true. They take a sample of 20 microwave ovens and find that they consume a mean of 257.3 W. The appropriate hypotheses to determine if the manufacturer's claim appears reasonable are:
    a. $H_0: \mu = 250$ versus $H_1: \mu \neq 250$
    b. $H_0: \mu \geq 250$ versus $H_1: \mu < 250$
    c. $H_0: \mu \leq 250$ versus $H_1: \mu > 250$
    d. $H_0: \mu \geq 257.3$ versus $H_1: \mu < 257.3$
    e. None of the above

Part 2:

1.

a. A study of the time spent shopping in a supermarket for a market basket of 20 specific items showed an approximately uniform distribution between 20 minutes and 40 minutes. What is the probability that the shopping time will be

i. less than 35 minutes? **(3 marks)**

ii. What is the standard deviation of the shopping time? **(3 marks)**

b. The speed in which the home page of a website is downloaded is an important quality characteristic of that website. Suppose that the mean time to download the home page for the Internal Revenue Service is 1.2 seconds. Suppose that the download time is normally distributed, with a standard deviation of 0.2 second. 99% of the download times are slower than how many seconds? **(4 marks)**

c. Customers arrive at the drive-up window of a fast-food restaurant at a rate of 2 per minute during the lunch hour. What is the probability that the next customer will arrive within 1 minute? **(3 marks)**

d. The amount of time a bank teller spends with each customer has a population mean, = 3.10 minutes and a standard deviation, = 0.40 minute. If you select a random sample of 16 customers,

i. what is the probability that the mean time spent per customer is at least 3 minutes? **(3 marks)**

ii. there is an 85% chance that the sample mean is less than how many minutes? **(3 marks)**

e. A political pollster is conducting an analysis of sample results in order to make prediction on election night. Assuming a two-candidate election, if a specific candidate receives at least 55% of the vote in the sample, that candidate will be forecast as the winner of the election. If you select a random sample of 100 voters, what is the probability that a candidate will be forecast as the winner when the population percentage of her vote is 50.1%.
(6 marks)

2.

a. A market researcher selects a simple random sample of customers from a population of 2 million customers. After analyzing the sample, she states that she has 95% confidence that the mean annual income of the 2 million customers is between $70,000 and $85,000. What is the value of the sample mean? **(3 marks)**

b. The telephone company has the business objective of wanting to estimate the proportion of households that would purchase an additional telephone line if it were made available at a substantially reduced installation cost. Data are collected from a random sample of 500 households. The results indicate that 135 of the households would purchase the additional telephone line at a reduced installation cost. Construct a 98% confidence interval estimate for the population proportion of households that would purchase the additional telephone line. **(7 marks)**

c. In the U.S. legal system, a defendant is presumed innocent until proven guilty. Consider a null hypothesis, that the defendant is innocent, and an alternative hypothesis, that the defendant is guilty. A jury has two possible decisions: Convict the defendant (i.e., reject the null hypothesis) or do not convict the defendant (i.e., do not reject the null hypothesis). Explain the meaning of the risks of committing either a Type I or Type II error in this example. **(4 marks)**

d.

i. The manager of a paint supply store wants to determine whether the mean amount of paint contained in 1-gallon cans purchased from a nationally known manufacturer is actually 1 gallon. You know from the manufacturer's specifications that the standard deviation of the amount of paint is 0.02 gallon. You select a random sample of 50 cans, and the mean amount of paint per 1-gallon can is 0.995 gallon. a. Is there evidence that the mean amount is different from 1.0 gallon? (**Use p-value approach and assume $\boldsymbol{\alpha = 0.01}$**). **(8 marks)**

ii. Construct a 99% confidence interval estimate of the population mean amount of paint.

(6 marks)

iii. Compare the conclusions in i and ii. **(2 marks)**

With Best Wishes

<u>Formula Sheet for Major 1</u>

$x!$

- $F(b) = P(X \le b), \;\; P(a \le x \le b) = F(b) - F(a)$
- $f(x) = \frac{1}{b-a}; \;\; F(x) = \frac{x-a}{b-a}; \;\; a \le x \le b; \;\; \mu = \frac{b+a}{2} \;\; \& \;\; \sigma = \sqrt{\frac{(b-a)^2}{12}}$
- $f(x) = \lambda e^{-\lambda x}, x > 0; \;\; F(x) = 1 - e^{-\lambda x}; \;\; \mu = \frac{1}{\lambda} \;\; \& \;\; \sigma = \frac{1}{\lambda}$
- $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$
- $Z = \frac{X-\mu}{\sigma} \;\; or \;\; Z = \frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{(\bar{X}-\mu)}{\frac{\sigma}{\sqrt{n}}}$

Confidence interval Estimation

- $\bar{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \;\; or \;\; \bar{x} \pm z_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}} \;\; or \;\; \bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}$
- $n = \left(\frac{z_{\frac{\alpha}{2}}\sigma}{e}\right)^2 \;\; or \;\; n = \left(\frac{z_{\frac{\alpha}{2}}s}{e}\right)^2$
- $(\bar{x}_1 - \bar{x}_2) \pm t_{\frac{\alpha}{2}, v} \; s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
- $n = \left(\frac{z_{\frac{\alpha}{2}}}{e}\right)^2 \hat{p}(1-\hat{p}) \;\; or \;\; n = \frac{1}{4}\left(\frac{z_{\frac{\alpha}{2}}}{e}\right)^2$
- $\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$