King Fahd University Of Petroleum & Minerals

Department Of Mathematics

STAT302 : Statistical Inference (231)

ID:________________

Name:____________________________

Question #	Full Mark	Marks Obtained
1	6	
2	6	
3	9	
4	10	
5	9	
6	10	
7	12	
8	10	
9	8	
10	10	
Total	90	

1. (4+2=6 Marks) The opening prices per share Y_1 and Y_2 of two similar stocks are independent random variables, each with a density function given by

$$f(y) = \frac{1}{2}e^{\frac{-(y-4)}{2}} \quad , y \geq 4$$

On a given morning, an investor is going to buy shares of whichever stock is less expensive. Find the

1. probability density function for the price per share that the investor will pay.

2. expected cost per share that the investor will pay.

2. (3+3=6 Marks) Suppose that Y_1 , Y_2 , . . . , Y_5 denotes a random sample from a uniform distribution defined on the interval (0, 1).

1. Find the density function for the second order statistic.

2. Find the joint density function for the second and fourth order statistics.

3. (3+6=9 Marks) Suppose that Y is normally distributed with mean 0 and unknown variance σ^2.

1. Show that $\dfrac{Y^2}{\sigma^2}$ is a pivotal quantity.

2. Find a 95% lower confidence limit for σ^2.

4. (4+6=10 Marks) Let Y have the probability density function

$$f(y;\theta) = \frac{2(\theta - y)}{\theta^2}\ , 0 < y < \theta \ and \ zero \ elsewhere$$

1. Show that $\frac{Y}{\theta}$ is a pivotal quantity.

2. Use the pivotal quantity $\frac{Y}{\theta}$ to find a 90% **upper** confidence limit for θ.

5. (3+3+3=9 Marks) The number of breakdowns per week for a type of minicomputer is a random variable Y with a Poisson distribution and mean λ. A random sample Y_1 , Y_2 , . . . , Y_n of observations on the weekly number of breakdowns is available. The weekly cost of repairing these breakdowns is C =3Y+Y^2.

1. Show that $\mu_c = E(C) = 4\lambda + \lambda^2$.

2. Show that $\hat{\theta} = 3\bar{Y} + \bar{Y}^2$ is a biased estimator of μ_c

3. Modify $\hat{\theta} = 3\bar{Y} + \bar{Y}^2$ to form an unbiased estimator of μ_c.

6. (4+4+2=10 Marks) Let $Y_1, Y_2, \ldots Y_n$ denote a sample of size n taken from exponential distribution with unknown parameter λ: $f(y;\lambda) = \lambda e^{-\lambda y}, \;\; y > 0$

1. Show that $2\lambda \sum_{1}^{n} Y_i$ is a pivotal quantity.

2. Use the pivotal quantity $2\lambda \sum_{1}^{n} Y_i$ to Find a $(1-\alpha)100\,\%$ lower confidence limit for λ.

3. If a sample of size n=7 yields average=4.77, use the result from part (b) to give a 95% lower confidence interval for λ.

7. (3+4+2+3=12 Marks) Let Y_1 , Y_2 , Y_3 be independent, exponentially distributed random variables with mean θ

1. Find the density function of $Y_{(1)} = Min(Y_1, Y_2, Y_3)$.

2. Show that $\hat{\theta} = Y_{(1)}$ is a biased estimator for θ, then find the MSE($\hat{\theta}$)

3. Find $\hat{\theta}'$, a multiple of $\hat{\theta} = Y_{(1)}$, that is an unbiased estimator for θ.

4. Is $\hat{\theta}'$ consistent. Why?

8. (3+7=10 Marks)

I. Studies of the effects of copper on a certain species of fish show the variance of measurements to be around 0.4 with concentration measurements in milligrams per liter. If n = 10 studies for copper are to be completed,

Find the probability that the sample mean will differ from the true population mean by no more than 0.5 .

- What assumption is needed.

II. Let S_1^2 denote the sample variance for a random sample of **n_1 =10** values for copper and let S_2^2 denote the sample variance for a random sample of **n_2 =8** values for lead, both samples using the same species of fish. The population variance for measurements on copper is assumed to be twice the corresponding population variance for measurements on lead.

Find **b** such that $P(\frac{S_1^2}{S_2^2} \leq b) = 0.95$ (Justify your work)

- What assumption(s) is (are) needed.

9. (4+4=8 Marks) Do you think that countries should pursue a program to send humans to Mars? An opinion poll indicated that 40% of the 1093 adults surveyed think that we should pursue such a program.

1. Estimate the proportion of all people who think that countries should pursue a program to send humans to Mars. Find a bound on the error of estimation.

2. Find the conservative bound of error that could be used for all sample proportions, with n=1093?

10. (10 Marks) Do SAT scores for higher school students differ depending on the students' intended field of study? Fifteen students who intended to major in engineering were compared with 15 students who intended to major in language and literature. Given in the accompanying table are the means and standard deviations of the scores on the verbal and mathematics portion of the SAT for the two groups of students:

Construct a 95% confidence interval for the difference in average verbal scores of students majoring in engineering and of those majoring in language. Interpret the interval. **What assumptions are necessary for the method used to be valid?**

	Verbal	Math
Engineering		
Mean	446	548
Std	42	57
Language		
Mean	534	517
Std	45	52