King Fahd University Of Petroleum & Minerals

Department Of Mathematics

STAT302 : Statistical Inference (231)

(Exam 2)

ID:_____

Name:_____

Question #	Full Mark	Marks Obtained
1	10	
2	25	
3	9	
4	5	
5	15	
6	12	
7	5	
8	9	
Total	90	

Question # 1. (5+5=10 pts) The geometric probability mass function is given by

$P(y|\theta) = \theta\,(1-\theta)^{(y-1)}$, for y = 1,2,3,. . . .

A random sample of size n is taken from a population with geometric distribution.

1- Find $\hat{\theta}_{MME}$, the method of moments estimator for θ.

2- Drive $\hat{\theta}_{MLE}$, the MLE for θ.

Question # 2. (5+5+5+10=25 pts) A random sample of size n is taken from the probability density function:

$$f(y|\theta) = \begin{cases} \frac{2\theta^2}{y^3}, & \theta < y < \infty \\ 0, & elsewhere \end{cases}$$

1) Show that $Y_{(1)}$ is a sufficient statistic for θ.

2) Find $\hat{\theta}_{MLE}$, the MLE for θ.

3) Derive $\hat{\theta}_{MME}$, the method of moments estimator for θ.

4) Find $\hat{\theta}_{MVUE}$. (the MVUE for θ)

Question # 3. (7+2=9 pts) Suppose that $Y_1, Y_2, \ldots, Y_n$ denotes a random sample from a population having an exponential distribution with mean θ.

a) Derive the most powerful test of size $\alpha = 0.05$ for $H_0 : \theta = \theta_0$ against $H_a : \theta = \theta_a$, where $\theta_a < \theta_0$.

b) Is the test derived in part (a) uniformly most powerful for testing $H_0: \theta = \theta_0$ against $H_a: \theta < \theta_0$? Explain.

Question # 4. (5 pts) A large-sample α-level test of hypothesis for

$$H_0: \theta = \theta_0 \text{ versus } H_1: \theta < \theta_0$$

rejects the null hypothesis if $\frac{\hat{\theta}-\theta_0}{\sigma_{\hat{\theta}}} < Z_{1-\alpha}$

Show that this is equivalent to rejecting H_0 if θ_0 is greater than the large-sample $100(1-\alpha)\%$ upper confidence bound for θ.

Question # 5. (5+5+5=15 pts) A manufacturer of hard safety hats for construction workers is concerned about the mean and the variation of the forces its helmets transmit to wearers when subjected to a standard external force. The manufacturer desires the mean force transmitted by helmets to be 800 pounds (or less), well under the legal 1000-pound limit, and desires σ to be less than 40. Tests were run on a random sample of $n = 40$ helmets, and the sample mean and variance were found to be equal to 825 pounds and 2350 pounds^2, respectively.

1. Do the data provide sufficient evidence to indicate that when subjected to the standard external force, the helmets transmit a mean force exceeding 800 pounds? **Use the p-value approach.**

2. How to conduct the test in part (1) using the confidence interval.

3. Do the data provide sufficient evidence to indicate that σ exceeds 40?

Question # 6. (5+7=12 pts) Measurements of anterior compartment pressure (in millimeters of mercury) were taken for ten healthy runners and ten healthy cyclists. The researchers also obtained pressure measurements for the runners and cyclists at maximal O_2 consumption.The data summary is given in the accompanying table.

	Runners		Cyclists	
condition	Mean	S	Mean	S
Rest	14.5	3.92	11.1	3.98
80% maximal O2 consumption	12.2	3.49	11.5	4.95
Maximal O_2 consumption	19.1	16.9	12.2	4.67

a. Is there sufficient evidence to support a claim that the variability of compartment pressure differs for runners and cyclists who are resting? What assumptions you need? Use $\alpha = .05$

b. Is there sufficient evidence to support a claim that the mean of compartment pressure differs for runners and cyclists who are resting? What assumptions you need? Use $\alpha = .05$

Question # 7. (5 pts) A random sample (1) of 37 second graders who participated in sports had manual dexterity scores with mean 32.19 and standard deviation 4.34. An independent sample (2) of 37 second graders who did not participate in sports had manual dexterity scores with mean 31.68 and standard deviation 4.56.

for testing $H_0: \mu_1 - \mu_2 = 0 \; vs \; H_1: \mu_1 - \mu_2 > 0$

Given: Test statistics =0.42 and **The rejection region** RR={Z>1.645}

Calculate β when $\mu_1 - \mu_2 = 3$.

Question # 8. (9 pts) A manufacturer of automatic washers offers a model in one of three colors: A, B, or C. Of the first 1000 washers sold, 400 were of color A. Would you conclude that customers have a preference for color A? Justify your answer.