Master

Stat 319 (211)
Major Ex-1.

1. A box of 500 rivets contains good rivets as well as rivets with defects summarized below: 30 rivets with type A defect, 15 rivets with type B defect, and 4 rivets with type A and type B defects. A rivet is chosen at random, what is the probability that it has exactly one defect?

a. 0.074
b. 0.082
c. 0.991
d. 0.098
e. 0.902

	A	A'	
B	4	11	15
B'	26	459	485
	30		500

$P(A \cap B') + P(A' \cap B)$

$26/500 + 11/500 = 37/500$

$= 0.074$

2. Suppose that a box contains 10% defective microchips. A purchaser decides to buy 5 microchips. Assume that the box has 30 microchips. What is the probability that the first two microchips in the sample will be defective and the last three will be good?

10 % of 30 = 3 defective chips

a. 0.00616
b. 0.0616
c. 0.616
d. 0.0729
e. 0.00729

$P(D_1 D_2 G_3 G_4 G_5) = 3/30 \cdot 2/29 \cdot 27/28 \cdot 26/27 \cdot 25/26$

$= 0.00616$

3. Suppose that B_1 and B_2 are mutually exclusive and complementary events, such that $P(B_1) = .6$ and Consider another event A such that $P(A \mid B_1) = .2$ and $P(A \mid B_2) = .5$. Find $P(B_1 \mid A)$.

$= \frac{P(B_1 \cap A)}{P(A)} = \frac{P(B_1) \cdot P(A|B_1)}{P(B_1 \cap A) + P(B_2 \cap A)}$

a. 0.375
b. 0.240
c. 0.800
d. 0.625
e. 0.200

$= \frac{P(B_1) \cdot P(A|B_1)}{P(B_1) \cdot P(A|B_1) + P(B_2) \cdot P(A|B_2)} = \frac{.6 \times .2}{.6 \times .2 + .4 \times .5}$

$= .375$

4. An encryption-decryption system consists of three elements: encode, transmit and decode. An encode error occurs in 0.9% of the message processed, transmission error occurs in 1% of the message, and the decode error occurs in 0.1% of the messages. Assume the errors are independent. What is the probability of the message with no error?

$P(\text{No Error}) = (1 - .009)(1 - .01)(1 - .001)$

$= .9801$

a. 0.9801
b. 0.0199
c. 9×10^{-8}
d. 0.9999
e. 0.991

5. Studies have shown that X, the number of refrigerators sold per week has the following probability distribution

x	0	2	4	5	10
$f(x)$	0.05	.30	0.45	0.18	0.02

The mean of X is equal to:

$\mu = E(X) = \sum x f(x)$

a. 3.5
b. 2.0
c. 4.0
d. 2.5
e. Cannot be found

$= 0 + .6 + 1.80 + .90 + 0.2$

$= 3.5$

6. We believe that 80% of the population of all Calculus I students consider calculus an exciting subject. Suppose we randomly and independently selected 24 students from the population. If the true percentage is really 80%, find the probability of observing 23 or more of the students who consider calculus to be an exciting subject.

$n = 24, \; p = 0.80$

$p(X \geq 23) = p(X=23) + p(X=24)$

a. 0.0330
b. 0.0047
c. 0.0283
d. 0.9669
e. 0.0283

$= \binom{24}{23}(.8)^{23}(.2)^{1} + \binom{24}{24}(.8)^{24}(.2)^{0}$

$= .0330$

7. An electronics store receives a shipment of 50 flat screen TVs of which 6 are defective. During the quality control inspection, 3 TVs are selected at random from the shipment for testing. The shipment will only be accepted if all 3 TVs pass the inspection. What is the probability that the shipment will be accepted?

$N = 50, \; n = 3, \; K = 44.$

a. 0.6757
b. 0.6815
c. 0.0017
d. 0.001
e. 0.083

$p(X=3) = \binom{44}{3}\binom{50-44}{3-3} \Big/ \binom{50}{3}$

$= 0.6757$

8. A specific automotive part that a service station stocks in its inventory has an 8% chance of being defective. Suppose many cars come into the service station needing this part each week. What is the probability that the fourth part retrieved from stock is the first defective?

$p = 0.08$

$P(X=4) = p(1-p)^{x-1}$

$= .08(.92)^{4-1} = 0.0623.$

a. 0.0623
b. 0.08
c. 0.0378
d. 0.0064
e. 0.3244

9. The manager of a movie theater has determined that the distribution of customers arriving at the concession stand is Poisson distributed with a standard deviation equal to 2 people per 10 minutes. If the servers can accommodate 3 customers in a 10-minute period, what is the probability that a customer will have to wait for service?

$\lambda = 4/10 \text{ minutes} = \mu = \delta^2$

$P(X>3) = 1-P(X\leq 3)$

$= 1-\left[\frac{4^0 e^{-4}}{0!} + \frac{4^1 e^{-4}}{1!} + \frac{4^2 e^{-4}}{2!} + \frac{4^3 e^{-4}}{3!}\right] = 1-.4335$

$= .5665$

a. 0.5665
b. 0.1804
c. 0.4335
d. 0.1954
e. 0.8046

10. Experience has shown that the width, in mm, of a certain type of plastic connector has the following probability density function

$$f(y) = \begin{cases} 50y & ,0.48 < y < 0.52 \\ 0 & ,otherwise \end{cases}$$

What is the probability that the width of the connector is no less than 0.51?

$\int_{.51}^{.52} 50y\,dy = 25((.52)^2 - (.51)^2)$

$= .2575$

a. 0.5150
b. 0.4983
c. 0.0431
d. 0.2575
e. 0

11. One study on managers' satisfaction with management tools reveals that 60% of all managers use self-directed work teams as a management tool. Suppose 70 managers selected randomly in the Kingdom of Saudi Arabia are interviewed. What is the probability that more than 37 use self-directed work teams as a management tool?

$n = 70, p = 0.60$

$\mu = np = 42$

$\delta = \sqrt{npq} = \sqrt{42(.4)}$

$= 4.0988$

$P(X > 37.5)$

$1-P\left(Z \leq \frac{37.5-42}{4.0988}\right)$

$= 1-.1357 = .8643$

a. 0.8643
b. 0.1357
c. 0.8888
d. 0.1112
e. 0.2869

12. The time between the arrivals of electronic messages at your computer is exponentially distributed with a mean of two hours. If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next three hours?

$\beta = 2$ hours. $p(X > 3$ hours$)$

$= 1 - p(X \le 3) = 1 - 1 + e^{-3/2} = e^{-1.5}$

$= 0.2231$

a. 0.2231
b. 0.0302
c. 0.9698
d. 0.7769
e. 0.1353

13. Assume that the life of a packaged magnetic disk exposed to corrosive gases has a Weibull distribution with β = 0.5 and the mean life is 600 hours. Determine the probability that a disk lasts at least 750 hours.

$\beta = 0.5$

Mean $= 600 = \delta \Gamma(1 + 1/\beta)$

$= \delta \Gamma(1 + 1/0.5)$

$\Rightarrow \delta = 600/2 = 300$

Now $p(X \ge 750)$

$= 1 - 1 + e^{-(x/\delta)^\beta}$

$= e^{-(750/300)^{0.5}} = e^{-1.5811} = 0.2057$

a. 0.2057
b. 0.0821
c. 0.9179
d. 0.7943
e. 0.3269

14. The following are the durations in minutes of a sample of long-distance phone calls made within the continental United States reported by one long-distance carrier.

Time (in Minutes)	Relative Frequency	(f)
0 but less than 5	0.37	37
5 but less than 10	0.22	22
10 but less than 15	0.15	15
15 but less than 20	0.10	10
20 but less than 25	0.07	7
25 but less than 30	0.07	7
30 or more	0.02	2
	1	100

$0.37 + 0.02 = 0.39$

$37 + 2 = 39$

If 100 calls were sampled, how many of them would have lasted less than 5 minutes or at least 30 minutes or more.

a. 39
b. 37
c. 35
d. Around 2
e. None of the above.

15. Health care issues are receiving much attention in both academic and political arenas. A sociologist recently conducted a survey of citizens over 60 years of age whose net worth is too high to qualify for Medicaid and have no private health insurance. The ages of 25 uninsured senior citizens were as follows:

Rank of $Q_3 = .75(26)$
$= 19.5$th value

60	61	62	63	64	65	66	68	68	69	70	73	73
74	75	76	76	81	81	82	86	87	89	90	92	

$Q_3 = \frac{81+82}{2} = 81.5$

Given: sum(x) = 1851, sum(x^2) = 139327

Standard Deviation (S.D) and the third Quartile of the ages of the uninsured senior citizens are:

$S = \sqrt{\frac{\sum x^2 - (\sum x)^2/n}{n-1}} = \sqrt{94.9567} = 9.7446$

a. S.D = 9.74 and Q3 = 81.5 years
b. S.D = 94.96 and Q3 = 65.5 years
c. S.D = 9.74 and Q3 = 73.0 years
d. S.D = 94.96 and Q3 = 74.4 years
e. S.D = 47.74 and Q3 = 16 years

16. The following frequency distribution shows the lifetimes of batteries (in hours).

Lifetime class	f	X	fX
[000, 100)	7	50	350
[100, 200)	18	150	2700
[200, 300)	12	250	3000
[300, 400)	9	350	3150
[400, 500)	4	450	1800
	50		11000

$\bar{X} = \frac{11000}{50} = 220$

Mode = 150

The approximate mean and mode lifetimes are, respectively:

a. 220 & 150 hours.
b. 250 & 300 hours.
c. 220 & 250 hours.
d. 300 & 200 hours.
e. 200 & 150 hours.