KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
MATHEMATICS DEPARTMENT

STAT 319 PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS
Semester 213, Second Exam
July 25th, 2022
Time allowed 90 minutes

Name: ____ ID #:________________________

Section #: ____ Serial #:________________________

Important Instructions:

Check that this exam has 18 questions

- All types of pagers or mobile phones are NOT allowed during the examination.
- Use HB 2.5 pencils only.
- Use a good eraser. DO NOT use the erasers attached to the pencil.
- Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
- When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
- The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
- When bubbling, make sure that the bubbled space is fully covered.
- When erasing a bubble, make sure that you do not leave any trace of penciling
- Formula sheet will be provided to you in exam. You are not allowed to bring, with you, formula sheet or any other printed/written paper.

1. A gas station operates two pumps, each can pump up to 10000 gallons of gas in a month. The total amount of gas pumped at the station in a month is a random variable (measures in 10000 gallons) with probability density function given by

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 < x < 2 \end{cases}$$

Given that the station pumped more than 10000 gallons in a particular month, find the probability that the station pumped more than 15000 gallons during the month

   a. 0.25
   b. 0.375
   c. 0.5
   d. 0.625
   e. 0.75

$$P(X > 1.5 | X > 1) = \frac{P(X > 1.5)}{P(X > 1)} = \frac{\int_{1.5}^{2}(2 - x)dx}{\int_{1}^{2}(2 - x)dx} = \frac{\left(2x - \frac{x^2}{2}\right)_{1.5}^{2}}{\left(2x - \frac{x^2}{2}\right)_{1}^{2}} = \frac{0.125}{0.5} = 0.25$$

2. A severe rainstorm is defined as a storm with 8 in. or more of rain. The cumulative distribution function of X as the random variable defining the amount of rain from a severe rainstorm is

$$F(x) = \begin{cases} 1 - \left(\frac{8}{x}\right)^4 & x \geq 8 \\ 0 & x < 8 \end{cases}$$

Street flooding occurs when rain exceeds 15 inches. Compute the probability of street flooding

   a. 0.0809
   b. 0.9101
   c. 0.4787
   d. 0.5213
   e. 0

$$P(X > 15) = 1 - P(X \leq 15) = 1 - F(15) = \left(\frac{8}{15}\right)^4 = 0.0809$$

3. The proportion X of time that an industrial robot is in operation during a 40 – hour week is a random variable given by

$$f(x) = \begin{cases} 2x & 0 \le x \le 1 \\ 0 & elsewhere \end{cases}$$

For the robot under study, the profit Y for a week is given by $Y = 200X - 60$, find the expected value of the profit in a week

a. 73.33
b. 300
c. 133.3
d. 140
e. 240

$$\mu_x = \int_0^1 x f(x)\,dx = \int_0^1 2x^2\,dx = \frac{2}{3}x^3\Big|_0^1 = \frac{2}{3}$$

$$\mu_y = 200\mu_x - 60 = 200\left(\frac{2}{3}\right) - 60 = 73.3$$

4. In Al Khobar, it is estimated that the maximum temperature in June is normally distributed with a mean of 43º and a standard deviation of 5°. Calculate the number of days in June (30 days) in which the temperature is expected to be between 41° and 47°.

a. 14 days
b. 10 days
c. 20 days
d. 5 days
e. 17 days

$$P(41 < X < 47) = P\left(\frac{41-43}{5} < Z < \frac{47-43}{5}\right) = P(-0.4 < Z < 0.8)$$
$$= P(Z < 0.8) - P(Z < -0.4) = 0.78814 - 0.34458 = 0.44356$$

$$\#\ of\ days = 0.44356(30) = 13.3 \approx 14$$

5. The reaction time of a driver to visual stimulus is normally distributed with a mean of 0.5 seconds and a standard deviation of 0.05 seconds. Determine the symmetrical bounds about the mean that include 90% of the time

a. 0.4175 and 0.5825
b. 0.4675 and 0.5325
c. 0.4360 and 0.5640
d. 0.4257 and 0.5635
e. 0.4038 and 0.5962

$$0.9 = P(\mu - a < X < \mu + a) = P\left(-\frac{a}{0.05} < Z < \frac{a}{0.05}\right)$$
$$0.05 = P\left(Z < -\frac{a}{0.05}\right)$$
$$\frac{a}{0.05} = 1.645$$
$$a = 0.0825 \quad seconds$$
$$0.5 - 0.0825 \quad and \quad 0.5 + 0.0825$$

6. An electronic office product contains 1000 electronic components. Assume that the probability that each component operates without failure during the useful life of the product is 0.9 and assume that the components fail independently. Approximate the probability that more than 109 of the original components fail during the useful life of the product.

a. 0.1587
b. 0.8413
c. 0.1706
d. 0.8289
e. 0.1867

$$P(X > 109) = P(X > 109.5) = P\left(Z > \frac{109.5 - 1000(01)}{\sqrt{1000(0.1)(0.9)}}\right) = P(Z > 1) = P(Z < -1)$$
$$= 0.15866$$

7. The time after reaching age 22 that it takes a student to graduate from KFUPM is approximately exponentially distributed with a mean of about 1.5 years. We are interested in the time after age 22 to graduation year. In a party of 500 students over age 24, how many students do you expect will not have graduated yet?

    a. 132
    b. 369
    c. 500
    d. 0
    e. 250

$$P(X > 2) = e^{-\frac{2}{1.5}} = 0.26359$$
$$\mu = np = 500(0.26359\ ) = 132$$

8. The life (in hours) of a magnetic resonance imaging machine (MRI) is modeled by a Weibull distribution with parameters β = 2 and δ = 500 hours. the median lifetime of the MRI is equal to

    a. 416.28 hours
    b. 240.23 hours
    c. 480.45 hours
    d. 232.56 hours
    e. We cannot tell

$$0.5 = e^{-\left(\frac{x}{500}\right)^2} \qquad x = 500(-\ln(0.5))^{0.5} = 8325.55$$

9. The lifetime (in hours) of a motor has a lognormal distribution with θ = 2 and ω = 1.2. What is the probability that the lifetime exceeds it mean?

   a. 0.27425
   b. 0.72575
   c. 0.99674
   d. 0.00236
   e. We cannot tell

$$P\left(e^W > e^{2+\frac{1.2^2}{2}}\right) = P(W > 2.72) = P(Z > 0.6) = P(Z < -0.6) = 0.27425$$

10. At a computer manufacturing company, the actual size of computer chips is normally distributed with a mean of 1 centimeter and a standard deviation of 0.1 centimeter. How many chips should be used in the sample if the sample mean is to be fewer than 0.99 centimeter with 99% assurance?

   a. 543 chips
   b. 100 chips
   c. 250 chips
   d. 30 chips
   e. We cannot tell

$$0.99 = P(\bar{X} < 0.99) = P\left(Z < \frac{(0.99-1)\sqrt{n}}{0.1}\right)$$

$$\frac{(0.99-1)\sqrt{n}}{0.1} = 2.33$$

$$\sqrt{n} = \frac{2.33}{-0.1}$$

$$n = \left(\frac{2.33}{-0.1}\right)^2 = 542.89$$

11. The lifetime of a certain type of battery is exponentially distributed with a mean of 10 hours and standard deviation 10 hours. If a randomly selected pack has 49 batteries, then what is the probability that the mean lifetime of this pack will exceed 11 hours?

    a. 0.24196
    b. 0.75804
    c. 0.33287
    d. 0.66713
    e. We cannot tell

$$P(\bar{X} > 11) = P\left(Z > \frac{(11-10)\sqrt{49}}{10}\right) = P(Z > 0.7) = P(Z < -0.7) = 0.24196$$

12. Assume that the average Systolic blood pressure for adults age 50-54 is 125 mmHg with a standard deviation of 5 mmHg. It is known that Systolic blood pressure is normally distributed. A sample of 25 adult Systolic blood pressure measurements are taken from the population. What is the probability that the average Systolic blood pressure will be less than 122 mmHg?

    a. 0.00135
    b. 0.99865
    c. 0.27425
    d. 0.72575
    e. 0.45224

$$P(\bar{X} < 122) = P\left(Z < \frac{(122-125)\sqrt{25}}{5}\right) = P(Z < -3) = 0.00135$$

13. Suppose that random samples of water from freshwater lake were taken and the calcium concentration (milligrams per 1000 liter) measured. A 95.45% confidence interval estimates on the mean calcium concentration is $3110 \le \mu \le 3230$, assume that the calcium concentration normally distributed with known variance, the length of a 97.22% confidence interval estimates the mean calcium concentration is

a. 132
b. 66
c. 30
d. 120
e. 128

$$1-\alpha = 0.9545 \quad \to \alpha = 0.0455 \quad \to \frac{\alpha}{2} = 0.02275 \ \to Z_{0.02275} = 2$$

$$e = \frac{l}{2} = \frac{120}{2} = 60 = z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} \ \to \ \frac{\sigma}{\sqrt{n}} = \frac{60}{2} = 30$$

$$1-\alpha = 0.9722 \quad \to \alpha = 0.0278 \quad \to \frac{\alpha}{2} = 0.0139 \ \to Z_{0.0139} = 2.2$$

$$l = 2e = 2z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} = 2(2.2)(30) = 132$$

14. Past experience has indicated that the breaking strength of yarn used in manufacturing drapery material is normally distributed with variance 16 psi^2. A random sample of nine specimens is tested, and the average breaking strength is found to be 98 psi. With what degree of confidence could we say that the mean breaking strength between 96 psi and 100 psi?

a. 87%
b. 90%
c. 95%
d. 85%
e. 99%

$$100 = 98 + z_{\frac{\alpha}{2}}\frac{\sqrt{16}}{\sqrt{9}}$$

$$2 = z_{\frac{\alpha}{2}}\frac{4}{3}$$

$$z_{\frac{\alpha}{2}} = \frac{2(3)}{4} = 1.5$$

$$z_{\frac{\alpha}{2}} = P(Z > 1.5) = \frac{\alpha}{2} \quad \to \quad P(Z < -1.5) = \frac{\alpha}{2}$$

$$\frac{\alpha}{2} = 0.06681 \quad \to \quad \alpha = 0.13362 \quad \to \quad 1-\alpha = 0.86638$$

15. The yield of a chemical process is being studied. From previous experience, yield is known to be normally distributed. The past five days of plant operation have resulted in the following percent yields: 91.6, 88.75, 90.8, 89.95, and 91.3. Find a 95% confidence interval on the true mean yield.

    a. $89.05 \leq \mu \leq 91.91$
    b. $89.16 \leq \mu \leq 91.80$
    c. $89.47 \leq \mu \leq 91.49$
    d. $89.38 \leq \mu \leq 91.58$
    e. $89.63 \leq \mu \leq 91.33$

$$\bar{x} = 90.48 \ \ and \ \ s = 1.1514$$
$$1 - \alpha = 0.95 \quad \rightarrow \alpha = 0.05 \quad \rightarrow \frac{\alpha}{2} = 0.025 \ \rightarrow \ t_{0.025,4} = 2.776$$
$$90.48 \pm 2.776\left(\frac{1.1514}{\sqrt{5}}\right)$$

16. The life in hours of a 75-watt light bulbs is known to be normally distributed with standard deviation 45 hours. we wanted the error in estimating the mean life from 90% confidence interval to be 5 hours. what sample size should be used?

    a. 220
    b. 312
    c. 133
    d. 440
    e. 538

$$n > \left(z_{0.05}\frac{\sigma}{e}\right)^2 = \left(1.645\frac{45}{5}\right)^2 = 219.18$$

17. The real estate assessor for a county government wishes to study various characteristics concerning single-family houses in the county. A random sample of 70 houses revealed 42 houses had central air conditioning. A 99% confidence interval for the proportion of houses that have central air conditioning given by

a. $0.449 \leq P \leq 0.751$
b. $0.503 \leq P \leq 0.696$
c. $0.485 \leq P \leq 0.715$
d. $0.599 \leq P \leq 0.601$
e. $0.542 \leq P \leq 0.658$

$$\frac{42}{70} \pm 2.575\sqrt{\frac{\frac{42}{70}\left(1-\frac{42}{70}\right)}{70}}$$
$$0.449 > P < 0.571$$

18. In a sample of 400 textile workers, 160 expressed dissatisfaction regarding a prospective plan to modify working conditions. Determine an optimal sample size to be 92% confident that the sample proportion differs from true proportion by less than 0.04

a. 460
b. 479
c. 434
d. 423
e. 406

$$n < \left(\frac{1.75}{0.04}\right)^2 \frac{160}{400}\left(1-\frac{160}{400}\right) = 459.37$$