KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
MATHEMATICS DEPARTMENT

STAT 319 PROBABILITY & STATISTICS FOR ENGINEERS & SCIENTISTS
Semester 213, Second Exam
August 10th, 2022
Time allowed 90 minutes

Name: ____ ID #:_________________________

Section #: ____ Serial #:_________________________

Important Instructions:

Check that this exam has 24 questions

- All types of pagers or mobile phones are NOT allowed during the examination.
- Use HB 2.5 pencils only.
- Use a good eraser. DO NOT use the erasers attached to the pencil.
- Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
- When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
- The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
- When bubbling, make sure that the bubbled space is fully covered.
- When erasing a bubble, make sure that you do not leave any trace of penciling
- Formula sheet will be provided to you in exam. You are not allowed to bring, with you, formula sheet or any other printed/written paper.

1. The number of contaminating particles on a silicon wafer prior to a certain rinsing process was determined for each wafer in a sample of size 91, resulting in the following frequencies

Number of particles	0	1	2	3	4	5	6	7	8	9	10
Frequency	1	2	3	9	11	15	19	10	12	4	5

The percentage of particles less than the mean

a. 45.05%
b. 50%
c. 54.95%
d. 42.90%
e. 57.1%

$$\bar{x} = \frac{520}{91} = 5.714$$
$$percentage = \frac{41}{91} = 0.4505$$

2. An oil company is bidding for the rights to drill a well in field A and a well in field B. The probability it will drill a well in field A is 40%. If it does, the probability the well will be successful is 45%. The probability it will drill a well in field B is 30%. If it does, the probability the well will be successful is 55%. Assuming that both fields are independent, the probability of both a successful well in field A and a successful well in field B is equal to

a. 0.0297
b. 0.18
c. 0.165
d. 0.12
e. 0.2475

$$P(success\ in\ A \cap success\ in\ B) = P(success\ in\ A)P(success\ in\ B) = (0.4)(0.5)(0.3)(0.55) = 0.0297$$

3. Let X be the damage incurred (in $) in a certain type of accident during a given year. Possible X values are 0, 1000, 5000, and 10000, with probabilities 0.8, 0.1, 0.08, and 0.02, respectively. A particular company offers a $500 deductible policy. What is the expected amount paid by the company?

   a. $200
   b. $300
   c. $100
   d. $700
   e. $500

X	0	1000	5000	10000	
P(X=x)	0.8	0.1	0.08	0.02	
E(x)	0	100	400	200	700

The expected amount paid by the company E(X-500) = 700-500 = 200

4. Suppose that the life length (in hours) of a certain radio tube is a random variable having a probability density function given by

$$f(x) = \frac{100}{x^2}, \qquad x \geq 100$$

What is the probability that if 3 such tubes are installed in set, exactly one will have to be replaced within the first 150 hours of service.

   a. $\frac{4}{9}$
   b. $\frac{1}{3}$
   c. $\frac{2}{3}$
   d. $\frac{5}{9}$
   e. $\frac{1}{2}$

$$P(X < 150) = P(100 < X < 150) = \int_{100}^{150} 100x^{-2}dx = \left(-\frac{100}{x}\right)_{100}^{150}$$

$$= \left(-\frac{100}{150} + \frac{100}{100}\right) = 1 - \frac{10}{15} = \frac{1}{3}$$

$$P(Y = 1) = C_1^3 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^2 = 3\left(\frac{1}{3}\right)\left(\frac{4}{9}\right) = \frac{4}{9}$$

5. In hypothesis testing, such as testing one population proportion, which of the following statements is correct when the significance level is 5%?

   a. Only statistics that occur with probability of less than 0.05 when H_0 is true are sufficient evidence to reject H_0.
   b. Only statistics that occur with probability of less than 0.95 when H_0 is true are sufficient evidence to reject H_0.
   c. Only statistics that occur with probability of 0.95 or more when H_0 is true are sufficient evidence to reject H_0.
   d. Only statistics that occur with probability of 0.05 or more when H_0 is true are sufficient evidence to reject H_0.
   e. We cannot tell.

6. Complete the statement by filling in the blanks.
   The null hypothesis H_0 is the statement of __________ and always has a _______ sign. The alternative hypothesis H_1 is the ___________ hypothesis. It is a statement about the value of a ___________ that we intend to test.

   a. no change; equal; research; parameter
   b. change; equal; no change; parameter
   c. no consequence; more than; research; sample
   d. no change; equal; research; sample
   e. change; equal; research; sample

7. A quality control manager thinks that there is a higher defective rate on the production line than the advertised value of 0.025. He does a hypothesis test with a significance level of 0.05. Symbolically, the null and alternative hypothesis are as follows:

$$H_0: P \leq 0.025 \quad vs \quad H_1: P > 0.025$$

He calculates a Type I error for the hypothesis test of defective light bulbs to be approximately 0.067, then

a. Probability of don't rejecting H_0, given that H_0 is true is 0.933.
b. Probability of Type II error is 0.933.
c. Probability of rejecting H_0, given that H_0 is true is 0.933.
d. Probability of rejecting H_1, given that H_0 is true is 0.933.
e. The p-value is equal to 0.067

8. An industrial engineer concerned with service at a large medical clinic recorded the duration of time from the time a patient called until a doctor or nurse returned the call. A sample of size 180 calls had a mean of 1.65 hours and a standard deviation of 0.82. We want to perform a test with the intention of establishing that the mean time to return a call is greater than 1.5 hours, at α = 0.05. What is the observed level of significance?

a. 0.0071
b. 0.9929
c. 0.4286
d. 0.5714
e. 0.05

$$P(Z > z_0) = 2P\left(Z > \frac{(1.65 - 1.5)\sqrt{180}}{0.82}\right) = P(Z > 2.45) = P(Z < -2.45) = 0.0071$$

9. A semiconductor manufacturer produces controllers used in automobile engine applications. The customer requires that the process fallout or fraction defective at a critical manufacturing step is less than 0.05 and that the manufacturer demonstrate process capability at this level of quality using α = 0.05. The semiconductor manufacturer takes a random sample of 200 devices and finds that four of them are defective. Can the manufacturer demonstrate process capability for the customer? Which of the following statements is true?

   a. The critical value is **-1.64** which is greater than the test statistic, we reject H_0 and conclude that the process fraction defective is less than 0.05. the process is capable
   b. The critical value is 1.96 which is less than the test statistic, we reject H_0 and conclude that the process fraction defective is less than 0.05. the process is capable
   c. The critical value is **-1.64** which is greater than the test statistic, we don't reject H_0 and conclude that the process fraction defective is not less than 0.05. the process is not capable
   d. The critical value is **1.64** which is greater than the test statistic, we don't reject H_0 and conclude that the process fraction defective is not less than 0.05. the process is not capable
   e. The critical value is -1.96 which is greater than the test statistic, we don't reject H_0 and conclude that the process fraction defective is not less than 0.05. the process is not capable

$\alpha = 0.05$ then the critical value $-z_{0.05} = -1.64$

we reject H_0 if $z_0 < -z_{0.05} = -1.64$ where

$$Z_0 = \frac{\frac{4}{200} - 0.05}{\sqrt{\frac{0.05(0.95)}{200}}} = -1.95$$

Since $Z_0 = -1.95 < -1.64$ reject H_0

10. A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed with standard deviation 0.25 volt, and the manufacturer wishes to test

$$H_0: \mu = 5 \quad vs \quad H_1: \mu \neq 5$$

For a sample of **size 9** units, if the non-rejection region is $4.85 \leq \bar{x} \leq 5.15$, find **the** type II error of the test for detecting a true mean output voltage of 5.1 volts.

   a. 0.72440
   b. 0.27560
   c. 0.07186
   d. 0.92814
   e. 0.72575

$$P(4.85 \leq \bar{x} \leq 5.15) = P(-3 \leq Z \leq 0.6) = P(Z \leq 0.6) - P(Z \leq -3)$$
$$= 0.72575 - 0.00135 = 0.7244$$

Regression models were used to analyze the data from a study investigating the relationship between the **macroscopic** magnetic relaxation time in crystal (in microseconds) (y) and the strength of the external biasing magnetic field (in kilogauss) (x). Summary quantities were

$$\sum_{i=1}^{12} x_i = 222, \sum_{i=1}^{12} y_i = 4116, \quad \sum_{i=1}^{12} x_i^2 = 4476, \quad \sum_{i=1}^{12} y_i^2 = 1538438, \quad \sum_{i=1}^{12} x_i y_i = 82788$$

Use the above information to answer the next 7 questions

11. What change in mean relaxation time would be expected for a one KG change in the filed strength?

   a. 18 microseconds
   b. 10 microseconds
   c. 12 microseconds
   d. 10.9 microseconds
   e. 18.5 microseconds

$$Sxx = 369 \quad Syy = 126650 \quad Sxy = 6642$$
$$\hat{\beta}_1 = \frac{Sxy}{Sxx} = 18$$

12. Suppose that the observed relaxation time when the filed strength is 18 KG, is 360 microseconds. The corresponding residual is

   a. 26 microseconds
   b. 334 microseconds
   c. 339 microseconds
   d. 10 microseconds
   e. 6472 microseconds

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 10$$
$$\hat{y}|_{x=18} = 10 + 18x = 336$$
$$error = 360 - 336 = 26$$

13. What proportion of the total variation in the relaxation time is explained by the linear regression model?

   a. 94.39%
   b. 5.60%
   c. 5.9%
   d. 94.1%
   e. 79.39%

$$R^2 = \frac{SSR}{SST} = \frac{Sxy^2}{SxxSyy} = 0.9439$$

14. The standard error of the estimate is

a. 26.63
b. 709.4
c. 1.922
d. 1.386
e. 3.696

$$S_e = \sqrt{\frac{SSE}{n-p}} = \frac{SST - SSR}{n-p} = \frac{Syy - \frac{Sxy^2}{Sxx}}{n-2} = 26.63$$

15. A 99% confidence interval estimate for the slope of the regression line is

a. Between 13.61 and 22.39
b. Between 13.69 and 22.31
c. Between 15.48 and 20.51
d. Between 15.51 and 20.49
e. Between 12.71 and 23.29

$$\hat{\beta}_1 \pm t_{0.005,10} S_e(\hat{\beta}_1)$$
$$18 \pm 3.1693 \frac{26.63}{\sqrt{369}}$$

16. The margin error of a 99% confidence interval for the expected relaxation time when the field strength is 18 kG?

a. 24.464
b. 4.3939
c. 7.7199
d. 27.731
e. 76.259

17. Suppose that a theoretical model suggests that the expected increase in relaxation time associated with a one KG change in the filed strength is 20 microseconds, the measured value of the test statistic is

a. − 1.44
b. 1.44
c. 12.98
d. − 12.98
e. − 0.259

$$T = \frac{\hat{\beta}_1 - \hat{\beta}_{10}}{S_e(\hat{\beta}_1)} = -1.44$$

A mechanical engineer is interested in studying the relationship between car mileage MPG (y) and five predictors, namely height (x1), horse power HP (x2), weight (x3), max load (x4), and turning circle (x5). Two regression models were fitted and the results are shown in the two outputs below:

Output 1

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value
Regression	5		200.815	58.28
Error		220.510		
Total	69	1224.585		

Model Summary

S	R-sq	R-sq(adj)	PRESS	R-sq(pred)
1.85619	81.99%		273.223	77.69%

Coefficients

Term	Coef	SE Coef	95% CI	T-Value	P-Value
Constant	52.48	4.84	(42.81, 62.14)	10.84	0.000
Height X1	-0.1326	0.0664	(-0.2652, 0.0000)	-2.00	0.050
HP X2	-0.01242	0.00531	(-0.02303, -0.00181)	-2.34	
Weight X3	-0.003625	0.000808	(-0.005239, -0.002010)	-4.49	0.000
Max Load X4	0.00524	0.00160	(0.00204, 0.00843)	3.27	0.002
Turning Circle X5	-0.309	0.113	(-0.535, -0.083)	-2.74	0.008

Output 2

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value
Regression		946.468		
Error	66			
Total	69	1224.585		

Use the above information to answer the next 7 questions

18. Using output 1, to test the significance of the HP (x2) to the MPG (y), the p-value of the test is

a. 0.02 < p – value < 0.05
b. 0.025 < p – value < 0.05
c. 0.005 < p value < 0.01
d. 0.0025 < p – value < 0.005
e. 0.05 < p – value < 0.1

$$p - value = 2P(T_{64} > |-2.34|) = 2P(T_{64} > 2.34)$$

$$0.02 < p - value < 0.05$$

19. Using output 1, at 2% level of significant, which of the following is true?

    a. Three predictors are significant in explaining the variation in the MPG.
    b. Two predictors are significant in explaining the variation in the MPG.
    c. One predictor is significant in explaining the variation in the MPG.
    d. Four predictors are significant in explaining the variation in the MPG.
    e. None of the predictors is significant in explaining the variation in the MPG.

Only x3, x4 and x5 are significant

20. Using output 1, the percentage of variation in the MPG (y) that can be explained by the variation in all of the predictors taking into account the given number of predictors and the given sample size, is equal to

    a. 80.59%
    b. 81.99%
    c. 77.69%
    d. 73.26%
    e. 86.56%

$$R_{adj}^2 = 1 - (1 - R^2) = 1 - (1 - R^2) = 0.8059$$

21. Using output 1, the predicted MPG (y) for a vehicle with a Height of 77, a HP 0f 320, a Weight of 5935, a Max load of 1460, and a Turning circle of 45, is equal to

    a. 10.526
    b. 15.206
    c. 16.520
    d. 12.506
    e. 11.778

$$\hat{y} = 10.526$$

22. Using output 2, the percentage of variation in the MPG (y) that can be explained by the variation in all of the predictors, is equal to

    a. 77.29%
    b. 81.99%
    c. 76.26%
    d. 73.26%
    e. 86.56%

$$R^2 = \frac{SSR}{SST} = 77.29$$

23. Using both outputs, the test statistic, for testing that X4 and X5 have significant contribution in explaining the variation in the MPG, is.

    a. 8.36
    b. 6.84
    c. 13.67
    d. 16.72
    e. 10.84

$$F_0 = \frac{\frac{SSR_{full} - SSR_{red}}{r}}{MSE_{full}} = \frac{\frac{1004.075 - 946.468}{2}}{\frac{22051}{64}} = 8.3598$$

24. Using both outputs, the degrees of freedom, for testing that X4 and X5 have significant contribution in explaining the variation in the MPG, are

    a. 2 and 64
    b. 3 and 64
    c. 2 and 66
    d. 3 and 66
    e. 2 and 69

$$df1 = k = 2 \quad and \; df2 = n - p = 64$$