Q1. (Example 4.2) Let the continuous random variable $X$ denote the diameter of a hole drilled in a sheet metal component. The target diameter is 12.5 millimeters. Most random disturbances to the process result in larger diameters. Historical data show that the distribution of $X$ can be modeled by a probability density function

$$f(x) = 20e^{-20(x-12.5)}, \qquad x \geq 12.5$$

If a part with a diameter greater than 12.60 mm is scrapped, what proportion of parts is scrapped?

A. 0.13533
B. 0.86467
C. 0.00004
D. 0.99996
E. 0.5

$$P(X > 12.6) = 1 - \int_{12.5}^{12.6} 20e^{-20(x-12.5)}\,dx = 0.13533$$

Q2 (Q4.48) The probability density function of the weight of packages delivered by a post office is

$$f(x) = \frac{70}{69x^2}, \quad 1 \leq x \leq 70$$

If the shipping cost is \$2.50 per pound, what is the average shipping cost of a package?

A. \$10.775
B. \$4.31
C. \$2.5
D. \$6.81
E. \$10.469

$$\mu_x = \int_1^{70} xf(x)\,dx = \int_1^{70} \frac{x70}{69x^2}\,dx = \int_1^{70} \frac{70}{69x}\,dx = \frac{70}{69}(\ln 70 - \ln 1) = 4.31$$

$$\mu_y = 2.5\mu_x = 2.5(4.31) = \$10.775$$

Q3: (4.62) The fill volume of an automated filling machine used for filling cans of carbonated beverage is normally distributed with a mean of 12.4 fluid ounces and a standard deviation of 0.1 fluid ounce. If all cans less than 12.3 or more than 12.5 ounces are scrapped, what proportion of cans is scrapped?

A. 0.3174
B. 0.6826
C. 0.1587
D. 0.8413
E. 0

X~ normal with $\mu_x = 12.4 \,\&\, \sigma = 0.1$

$$\begin{aligned} P(scrapped) &= P(X < 12.3) + P(X > 12.5) \\ &= P\left(Z < \frac{12.3 - 12.4}{0.1}\right) + P\left(X > \frac{12.5 - 12.4}{0.1}\right) \\ &= P(Z < -1) + P(X > 1) = 2P(Z < -1) = 2(0.1587) = 0.3174 \end{aligned}$$

Q4: (Q4.96) The life of automobile voltage regulators has an exponential distribution with a mean life of six years. You purchase a six-year-old automobile, with a working voltage regulator and plan to own it for six years. What is the probability that the voltage regulator fails during your ownership?

A. 0.6321
B. 0.3678
C. 0.5
D. 0.5654
E. 0.4346

X~ exponential with $\mu_x = 6$

$$P(fails\ during\ the\ ownship) = P(X < 6) = 1 - e^{-1} = 0.6321$$

Q5: (Q4.129) The life (in hours) of a computer processing unit (CPU) is modeled by a Weibull distribution with parameters β = 2 and δ = 900 hours. What is the probability that the CPU fails before the mean life of the CPU?

A. 0.5441
B. 0.4559
C. 0.6099
D. 0.3901
E. 0.5000

X~ Weibull with β = 2 and δ = 900

$$\mu_x = \delta\Gamma\left(1+\frac{1}{\beta}\right) = 900\Gamma\left(1+\frac{1}{2}\right) = 900(0.5)\Gamma\left(\frac{1}{2}\right) = 900(0.5)\sqrt{\pi}$$

$$P(X < \mu_x) = 1 - e^{-\left(\frac{900(0.5)\sqrt{\pi}}{900}\right)^2} = 0.5441$$

Q6: (Q4.141) The length of time (in seconds) that a user views a page on a Web site before moving to another page is a lognormal random variable with parameters $\theta = 0.5$ & $\omega^2 = 2$. By what length of time have 50% of the users moved to another page?

A. 1.649 seconds
B. 1 second
C. 6.782 seconds
D. 2.495 seconds
E. 0.147 seconds

X~ lognormal with $\theta = 0.5$ & $\omega^2 = 2$

$$0.5 = P(X < a) = P(W < \ln(a)) = P\left(Z < \frac{\ln(a) - 0.5}{2}\right)$$

$$\frac{\ln(a) - 0.5}{2} = 0 \quad \rightarrow \quad \ln(a) = 0.5 \quad \rightarrow a = e^{0.5} = 1.649$$

Q7: (Example 7.1 & Q7.9) An electronics company manufactures resistors that have a mean resistance of 100 ohms and standard deviation of 10 ohms. The distribution of resistance is normal. How large must the random sample be if you want the standard error of the sample average to be no more than 1.5 ohms?

A. 45
B. 7
C. 35
D. 1
E. 350

X~ normal with $\mu_x = 100$ & $\sigma = 10$

$\bar{X}$ ~ normal with $\mu_{\bar{X}} = 100$ & $\sigma_{\bar{X}} = \frac{10}{\sqrt{n}}$

$$\sigma_{\bar{X}} = \frac{10}{\sqrt{n}} = 1.5 \quad \rightarrow \quad n = \frac{10}{1.5}\left(\frac{10}{1.5}\right)^2 = 44.44 \approx 45$$

Q8: (Example 7.1) An electronics company manufactures resistors that have a mean resistance of 100 ohms and a standard deviation of 10 ohms. The distribution of resistance is normal. Find the probability that a random sample of size 25 resistors will have an average resistance of fewer than 95 ohms.

A. 0.0062
B. 0.9938
C. 0.2776
D. 0.6914
E. 0.5000

X~ normal with $\mu_x = 100$ & $\sigma = 10$

$\bar{X}$ ~ normal with $\mu_{\bar{X}} = 100$ & $\sigma_{\bar{X}} = \frac{10}{\sqrt{25}} = 2$

$$P(\bar{X} < 95) = P\left(Z < \frac{95 - 100}{2}\right) = P(Z < -2.5) = 0.0062$$

Q9: A machine filling mini boxes of rasins fills the boxes so that weight of the boxes has a population mean 14.1 grams and a population standard deviation 1.4 grams. A random sample of 49 boxes is selected. What is the probability that the total weight of the rasins will be at most 675 grams?

A. 0.0526
B. 0.4090
C. 0.9438
D. 0.5910
E. 0.5

X~ a distribution with $\mu_x = 14.1 \ \& \ \sigma = 1.4$

Since n large, by C.L.T $\bar{X}$ ~ approximately normal with $\mu_{\bar{X}} = 1.1 \ \& \ \sigma_{\bar{X}} = \frac{1.4}{\sqrt{49}}$

$$P\left(\sum x < 675\right) = P\left(\bar{X} < \frac{675}{49}\right) = P\left(Z < \frac{\frac{675}{49} - 14.1}{\frac{1.4}{\sqrt{49}}}\right) = P(Z < -1.62) = 0.0526$$

Q10: (Q7.16) Scientists at the Hopkins Memorial Forest in western Massachusetts have been collecting meteorological and environmental data in the forest data for more than 100 years. In the past few years, sulfate content in water samples from Birch Brook has averaged 7.48 mg/L with a standard deviation of 1.60 mg/L. The standard error of the sulfate in a collection of 10 water samples?

A. 0.5059 mg/L
B. 1.6 mg/L
C. 31.623 mg/L
D. 0.1834 mg/L
E. 7.48 mg/L

X~ normal with $\mu_x = 7.48 \ \& \ \sigma = 1.6$

Since n large, by C.L.T $\bar{X}$ ~ approximately normal with $\mu_{\bar{X}} = 7.48 \ \& \ \sigma_{\bar{X}} = \frac{1.6}{\sqrt{10}} = 0.5059$

Q11: (Example 7.2) Suppose that the random variable X has the continuous uniform distribution with probability density function given by

$$f(x) = 0.5, \quad 4 \le x \le 6$$

How many observations should be selected in a sample from this distribution, if the sample mean is to be fewer than 5.2 with 99% assurance?

A. 46
B. 7
C. 25
D. 5
E. 30

X~ uniform over (4,6) with $\mu_x = 5 \,\&\, \sigma^2 = \frac{4}{12} = \frac{1}{3}$

$\bar{X}$ ~ approximately normal with $\mu_{\bar{X}} = 5 \& \sigma_{\bar{X}} = \frac{\sqrt{\frac{1}{3}}}{\sqrt{n}} = \sqrt{\frac{1}{3n}}$

$$0.99 = P(\bar{X} < 5.2) = P\left(Z < \frac{5.2-5}{\sqrt{\frac{1}{3n}}}\right)$$

$$\frac{5.2-5}{\sqrt{\frac{1}{3n}}} = 2.33 \;\rightarrow\; \sqrt{\frac{1}{3n}} = \frac{0.2}{2.33} \;\rightarrow\; \frac{1}{3n} = \left(\frac{0.2}{2.33}\right)^2 \;\rightarrow n = \frac{\left(\frac{233}{20}\right)^2}{3} = 46$$

Q12: (page 239) For s sample of size 30 selected randomly from a lognormal distribution with $\theta = 2 \,\&\, \omega = 0.75$, the sampling distribution of the sample mean will be

A. Approximately normal with standard error 1.55
B. Approximately normal with standard error 8.51
C. lognormal with standard error 8.51
D. lognormal with standard error 1.55
E. normal with standard error 8.51

X~ lognormal over (4,6) with $\theta = 2 \,\&\, \omega = 0.75$

Since n large, $\bar{X}$ ~ approximately normal with $\mu_{\bar{X}} = e^{2+\frac{0.75^2}{2}}$ $\& \, \sigma_{\bar{X}} = \frac{\sqrt{e^{2(2)+0.75^2}\left(e^{0.75^2}-1\right)}}{\sqrt{30}} = 1.55$

Q13: (Q8.1-c) For a normal population with known variance, the confidence level for the interval $\bar{x} \pm 1.85\frac{\sigma}{\sqrt{n}}$ is

A. 93.56%
B. 96.78%
C. 98.39%
D. 95%
E. 90%

$$Z_{\frac{\alpha}{2}} = P(Z > 1.85) = P(Z < -1.85) = \frac{\alpha}{2}$$
$$\frac{\alpha}{2} = 0.0322 \ \rightarrow \ \alpha = 0.0644 \ \rightarrow \ 1 - \alpha = 0.9356$$

Q14: (Q8.9) Suppose that 100 random samples of water from a freshwater lake were taken and the calcium concentration (milligrams per liter) measured. A 95% CI on the mean calcium concentration is $0.49 \leq \mu \leq 0.82$. the length of a 99% CI calculated from the same sample data be

A. wider
B. shorter
C. same length
D. sometimes wider and sometimes shorter
E. we need more information to tell.

$1 - \alpha$ increase, the margin error increases the interval will be wider

Q15: (Q8.18) Past experience has indicated that the breaking strength of yarn used in manufacturing drapery material is normally distributed with standard deviation of 2 psi. A random sample of nine specimens is tested, and the average breaking strength is found to be 98 psi. the length of a 95% confidence interval on the true mean breaking strength given by

A. 2.613
B. 1.306
C. 1.1
D. 2.2
E. 1.706

$$\ell = 2e = 2Z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} = 2(1.96)\frac{2}{\sqrt{9}} = 2.613$$

Q16: (Q8.11) The yield of a chemical process is being studied. From previous experience, yield is known to be normally distributed with standard deviation 3. How large must n be if the length of the 99% CI is to be 1?

A. 239
B. 60
C. 35
D. 139
E. 98

$$n = \left(\frac{Z_{\frac{\alpha}{2}}\sigma}{e}\right)^2 = \left(\frac{2.575(3)}{0.5}\right)^2 = 239$$

Q17: (Example 8.8) In a random sample of 85 automobile engine crankshaft bearings, 10 have a surface finish that is rougher than the specifications allow. A 95% confidence interval estimate of the proportion of bearings in the population that exceeds the roughness specification is given by

A. Between 4.92% and 18.61%
B. Between 6.02% and 17.51%
C. Between 7.29% and 16.24%
D. Between 2.77% and 20.76%
E. Between 8.44% and 15.08%

$$\hat{p} \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\frac{10}{85} \pm 1.96\sqrt{\frac{\frac{10}{85}\left(1-\frac{10}{85}\right)}{85}}$$

$$0.0492 \leq p \leq 0.1861$$

Q18: (Example 8.9) What is the needed sample size to estimate the true population proportion if we wanted to be at least 95% confident that our estimate of the true population proportion was within 0.05 regardless of the true value of the population proportion p

A. 385
B. 271
C. 666
D. 164
E. 256

$$n = \left(\frac{1.96}{0.05}\right)^2 \frac{1}{4} = 385$$