King Fahd University of Petroleum and Minerals
Department of Mathematics

**Stat 319**

**Major Exam II**

**241**

**November 04, 2024**

**Net Time Allowed: 90 Minutes**

# USE THIS AS A TEMPLATE

**Write your questions, once you are satisfied upload this file.**

1. (Q: 4-9(b), page 139) The probability density function of the net weight in kg of a packaged chemical herbicide is $f(x) = \frac{1}{9}$ for $15.5 < x < 24.5$ kg. How much chemical is contained in 90% of all packages, i.e., the weight above which 90% of the packages lie?

   (a) 16.4 kg
   (b) 23.6 kg
   (c) 20 kg
   (d) 8.1 kg
   (e) 15.6 kg

2. (Q: 4-23, page 140) The gap width is an important property of a magnetic recording head. In coded units, if the width is a continuous random variable over the range from $0 < x < 0.5$ with $f(x) = 8x$, determine the cumulative distribution function of the gap width.

   (a) $F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 4x^2 & \text{for } 0 < x < 0.5 \\ 1 & \text{for } x \geq 0.5 \end{cases}$

   (b) $F(x) = \begin{cases} 0 & \text{for } x \leq 0.5 \\ 0.5x^2 & \text{for } 0.5 < x < 8 \\ 4x^2 & \text{for } x \geq 8 \end{cases}$

   (c) $F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ 0.5x^2 & \text{for } 0 < x < 0.5 \\ 1 & \text{for } x \geq 0.5 \end{cases}$

   (d) $F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x^2 & \text{for } 0 < x < 0.25 \\ 8x^2 & \text{for } x \geq 0.25 \end{cases}$

   (e) $F(x) = \begin{cases} 0 & \text{for } 0 \leq x \leq 0.5 \\ 8x^2 & \text{for } 0.5 < x < 8 \\ 1 & \text{for } x \geq 8 \end{cases}$

3. (Q: 4-73, page 143) Assume that a random variable is normally distributed with a mean of 54 and a standard deviation of 2.5. Consider an interval of length two units that starts at the value $a$, so that the interval is $[a, a+2]$. For what value of $a$ is the probability of the interval greatest?

   (a) 53
   (b) 52
   (c) 54
   (d) 55
   (e) 56

4. (Q: 4-103(e), page 146) The lifetime of a mechanical assembly in a vibration test is exponentially distributed with a mean of 500 hours. If 10 assemblies are tested, what is the probability that all have failed by 810 hours? Assume that the assemblies fail independently.

   (a) 0.1102
   (b) 0.8021
   (c) 0.8898
   (d) 0.1979
   (e) 0

5. (Q: 4-36(b), page 140) The probability density function of the weight of packages delivered by a post office is $f(x) = \frac{50}{49x^2}$ for $1 < x < 50$ kg. If the shipping cost is \$2.50 per kg, what is the average shipping cost of a package?

   (a) \$9.98

   (b) \$3.99

   (c) \$1.02

   (d) \$2.55

   (e) \$125

6. (Q: 4-131(c), page 147) The life (in hours) of a magnetic resonance imaging machine (MRI) is modeled by a Weibull distribution with parameters $\beta = 2$ and $\delta = 500$ hours. Determine the probability that the MRI fails before 320 hours.

   (a) 0.336

   (b) 0.506

   (c) 0.664

   (d) 1.634

   (e) 0.913

7. (Q: 4-169(c), page 150) Suppose that $X$ has a lognormal distribution and that the mean ($\mu$) and variance ($\sigma^2$) of $X$ are 36 and 3000, respectively. Determine the probability that $X$ is less than 100.

   (a) 0.93056
   (b) 0.97558
   (c) 0.28434
   (d) 0.87286
   (e) 0.87900

8. (Q: 7-7, page 260) The compressive strength of concrete is normally distributed with $\mu = 17200\,\text{KN/m}^2$ and $\sigma = 345\,\text{KN/m}^2$. Find the probability that a random sample of $n = 5$ specimens will have a sample mean strength that falls in the interval from 17217 $\text{KN/m}^2$ to 17325 $\text{KN/m}^2$.

   (a) 0.24723
   (b) 0.54380
   (c) 0.79103
   (d) 0.12064
   (e) 0.36614

9. (Q: 7-6, page 260) A synthetic fiber used in manufacturing carpet has tensile strength that is normally distributed with mean 520 $KN/m^2$ and standard deviation 25 $KN/m^2$. A random sample of $n = 6$ fiber specimens is selected and the standard deviation of the sample mean ($\sigma_{\bar{X}}$) is computed. How is the standard deviation of the sample mean changed when the sample size is increased from $n = 6$ to $n = 49$?

   (a) The standard deviation of the sample mean will decrease.
   (b) The standard deviation of the sample mean will increase.
   (c) The standard deviation of the sample mean will not change.
   (d) The standard deviation of the sample mean will become undefined.
   (e) The standard deviation of the sample mean will equal the population standard deviation.

10. (Q: 8-9, page 291) Suppose that $n = 100$ random samples of water from a freshwater lake were taken and the calcium concentration (mg/L) measured. A 95% Confidence Interval (CI) on the mean calcium concentration is $0.52 \leq \mu \leq 0.74$. Which one of the following statements is not true?

    (a) There is a 95% chance that $\mu$ is between 0.52 and 0.74.
    (b) A 99% CI calculated from the same sample data will be wider than 0.52 to 0.74.
    (c) If $n = 100$ random samples of water from the lake were taken and the 95% CI on $\mu$ computed, and this process were repeated 1000 times, 950 of the CIs would contain the true value of $\mu$.
    (d) A 90% CI calculated from the same sample data will be narrower than 0.52 to 0.74.
    (e) If the sample size was increased, the width of the 95% CI would likely decrease, assuming the variability in the data stays the same.

11. (Q: 8-16, page 291) The life in hours of a 75-watt light bulb is known to be normally distributed with $\sigma = 39$ hours. Suppose that you wanted the total width of the two-sided confidence interval on mean life to be 18 hours at 95% confidence (i.e. $Z_{\alpha/2} = 1.96$). What sample size should be used?

    (a) 73
    (b) 72
    (c) 18
    (d) 19
    (e) 5

12. (Q: 8-35, page 292) The brightness of a television picture tube can be evaluated by measuring the amount of current required to achieve a particular brightness level. A sample of 5 tubes results in $\bar{x} = 315.5$ and $s = 15.3$. Find (in microamps) a 99% (two-sided) confidence interval on mean current required. What is the lower limit of your confidence interval?

    (a) 283.9977
    (b) 347.0023
    (c) 287.9116
    (d) 343.0884
    (e) 289.8616

13. (Q: 8-96, page 298) A manufacturer of electronic calculators takes a random sample of 120 calculators and finds nine defective units. Construct a 95% (two-sided) confidence interval on the population proportion. The upper confidence limit is given as:

(a) 0.1221
(b) 0.0130
(c) 0.1370
(d) 0.0761
(e) 0.2471

14. (Q: 8-58, page 295) A random sample of 90 suspension helmets used by motorcycle riders and automobile race-car drivers was subjected to an impact test, and some damage was observed on 18 of these helmets. Using the point estimate of $p$ from the 90 helmets, how many helmets must be tested to be 97% confident that the error in estimating $p$ is less than 0.02?

(a) 1884
(b) 18
(c) 2944
(d) 1537
(e) 2943