King Fahd University of Petroleum and Minerals
Department of Mathematics

Stat 319

Final Exam

242

12 May 2025

Net Time Allowed: 120

MASTER VERSION

1. Consider the computer output given here. What is the estimated value of σ^2?

The regression equation is: $\hat{y} = 12.857 + 2.3445\ x$

Predictor	Coef	SE Coef	T	$p-$value
Constant	12.857	1.032	?	?
X	2.3445	0.115	?	?

$S = 1.48114$ $R^2 = 98.1\%$ $R^2_{\text{adj}} = 97.9\%$

Analysis of Variance

Source	*df*	SS	MS	F	$p-$value
Regression	1	912.43	912.43	?	?
Residual Error	8	17.55	?		
Total	9	929.98			

(a) 2.1938 ____(correct)
(b) 17.5500
(c) 2.3445
(d) 20.3870
(e) 1.0320

2. Consider the computer output given here. We are interested in testing the significance of intercept. What is the value of the test statistic?

The regression equation is: $\hat{y} = 12.857 + 2.3445\ x$

Predictor	Coef	SE Coef	T	$p-$value
Constant	12.857	1.032	?	?
X	2.3445	0.115	?	?

$S = 1.48114$ $R^2 = 98.1\%$ $R^2_{\text{adj}} = 97.9\%$

Analysis of Variance

Source	*df*	SS	MS	F	$p-$value
Regression	1	912.43	912.43	?	?
Residual Error	8	17.55	?		
Total	9	929.98			

(a) 12.4583 ____(correct)
(b) 12.8570
(c) 912.4300
(d) 1.4811
(e) 1.0320

3. The "cold start ignition time" of an automobile engine is being investigated by a gasoline manufacturer. The following times (in seconds) were obtained for a test vehicle: 1.75, 1.92, 2.62, 2.35, 3.09, 3.15, 2.53, 1.91. What is the value of inter quartile range of the data?

   (a) 1.0600 ______________________________ (correct)
   (b) 2.9725
   (c) 1.9125
   (d) 0.5900
   (e) 2.6850

4. For the copper current measurement, the cumulative distribution function of continuous random variable X is given as:

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.05x, & 0 \leq x < 20 \\ 1, & x \geq 20 \end{cases}$$

   What is the probability that a current measurement is between 5 and 20 milliamperes?

   (a) 0.750 ______________________________ (correct)
   (b) 0.500
   (c) 0.050
   (d) 0.075
   (e) 0.150

5. Suppose that the random variable X has the continuous distribution given by the following:

$$f(x) = \begin{cases} 1, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Suppose that a random sample of $n = 12$ is selected from this distribution. What is the sampling distribution of $\bar{X} + 6$?

(a) Approximately $N\left(\mu_{\bar{x}} = \frac{13}{2}, \sigma_{\bar{x}} = \frac{1}{12}\right)$, by central limit theorem ________(correct)
(b) Approximately $N\left(\mu_{\bar{x}} = \frac{13}{2}, \sigma_{\bar{x}} = \frac{73}{12}\right)$, by central limit theorem
(c) Approximately $N\left(\mu_{\bar{x}} = \frac{1}{2}, \sigma_{\bar{x}} = \frac{1}{12}\right)$, by central limit theorem ---Note:Option c is also considered correct in grading
(d) Exactly $N\left(\mu_{\bar{x}} = \frac{13}{2}, \sigma_{\bar{x}} = \frac{1}{12}\right)$
(e) Approximately $N\left(\mu_{\bar{x}} = \frac{13}{2}, \sigma_{\bar{x}} = \frac{1}{\sqrt{12}}\right)$, by central limit theorem

6. A random sample has been taken from a normal distribution and the following confidence intervals for the population mean (μ) are constructed using the same data: (37.53, 49.87) and (35.59, 51.81). What is the value of the sample mean?

(a) 43.70 ________(correct)
(b) 87.40
(c) 37.53
(d) 35.59
(e) 21.85

7. A textile fiber manufacturer is investigating a new drapery yarn, which the company claims has a mean thread elongation of 12 kilograms with a standard deviation of 0.5 kilograms. The company wishes to test the hypothesis $H_0 : \mu = 12$ against $H_1 : \mu < 12$, using a random sample of four specimens. What is the type I error probability if the critical region is defined as $\bar{x} < 11.5$ kilograms?

   (a) 0.02275 ________________ (correct)
   (b) 0.97725
   (c) 0.15866
   (d) 0.84134
   (e) 0.05000

8. A textile fiber manufacturer is investigating a new drapery yarn, which the company claims has a mean thread elongation of 12 kilograms with a standard deviation of 0.5 kilograms. The company wishes to test the hypothesis $H_0 : \mu = 12$ against $H_1 : \mu < 12$, using a random sample of four specimens. Calculate the P-value if the observed statistic is $\bar{x} = 11.25$.

   (a) 0.00135 ________________ (correct)
   (b) 0.99865
   (c) 0.00270
   (d) 0.99730
   (e) 0.05000

9. A manufacturer is interested in the output voltage of a power supply used in a PC. Output voltage is assumed to be normally distributed, with standard deviation 0.25 volt, and the manufacturer wishes to test $H_0 : \mu = 5$ against $H_1 : \mu \neq 5$, using $n = 8$ units. The acceptance region is $4.85 \leq \bar{x} \leq 5.15$. Find the power of the test for detecting a true mean output voltage of 5.1 volts.

   (a) 0.2867 ____(correct)
   (b) 0.7133
   (c) 0.0895
   (d) 0.9105
   (e) 0.9500

10. Medical researchers have developed a new artificial heart constructed primarily of titanium and plastic. The heart will last and operate almost indefinitely once it is implanted in the patient's body, but the battery pack needs to be recharged about every four hours. A random sample of 50 battery packs is selected and subjected to a life test. The average life of these batteries is 4.05 hours. Assume that battery life is normally distributed with standard deviation $\sigma = 0.2$ hour. We are interested in testing the claim that mean battery life exceeds 4 hours, using $\alpha = 0.0044$. We reject the null hypothesis if the calculated value of test statistic is:

    (a) greater than 2.62. ____(correct)
    (b) less than -2.62.
    (c) greater than 2.85.
    (d) less than 2.62.
    (e) greater than 1.96.

11. The advertised claim for batteries for cell phones is set at 48 operating hours, with proper charging procedures. A study of 500 batteries is carried out and 45 stop operating prior to 48 hours. We want to test if these experimental results support the claim that less than 12 percent of the company's batteries will fail during the advertised time period, with proper charging procedures. What is value of the test statistic?

    (a) -2.06 ________ (correct)
    (b) 2.77
    (c) -4.13
    (d) 2.34
    (e) -0.65

12. Suppose data is obtained from 20 pairs of (x, y) and the sample correlation coefficient is 0.4438. We want to test the hypothesis $H_0 : \rho = 0$ against $H_1 : \rho \neq 0$ with $\alpha = 0.01$. What is the approximate $p-$value of this test?

    (a) 0.050 ________ (correct)
    (b) 0.025
    (c) 0.010
    (d) 0.005
    (e) 0.020

13. If A, B, and C are mutually exclusive events with $P(A) = 0.2$, $P(B) = 0.3$, and $P(C) = 0.4$. Determine $P(A' \cap B' \cap C')$.

    (a) 0.100 ____(correct)
    (b) 0.000
    (c) 0.900
    (d) 0.024
    (e) 0.976

14. Two new product designs are to be compared on the basis of revenue potential. Marketing feels that the revenue from design A can be predicted quite accurately to be \$3 million. The revenue potential of design B is more difficult to assess. Marketing concludes that there is a probability of 0.3 that the revenue from design B will be \$7 million, but there is a 0.7 probability that the revenue will be only \$2 million. What is the standard deviation value of the revenue from design B (in millions of dollars)?

    (a) 2.29 ____(correct)
    (b) 3.50
    (c) 3.00
    (d) 5.25
    (e) 6.50

15. Consider the computer output given here. What is the approximate p-value of $F-$test for testing the significance of regression?

The regression equation is: $\hat{y} = 2.64 + 2.233\ x$

Predictor	Coef	SE Coef	T	$p-$value
Constant	2.64	3.74	0.71	0.488
X	2.233	0.183	12.22	?

$S = 4.74778$ $R^2 = 87.15\%$ $R^2_{\text{adj}} = 86.57\%$

Analysis of Variance

Source	*df*	SS	MS	F	$p-$value
Regression	1	3364.1	3364.1	149.24	?
Residual Error	22	495.9	22.54		
Total	23	3860.1			

(a) $p-$ value < 0.005 ________________ (correct)

(b) $0.005 < p -$ value < 0.01

(c) $0.01 < p -$ value < 0.025

(d) $0.025 < p -$ value < 0.05

(e) $p -$ value > 0.25

16. You have fit a regression model with two regressors to a data set that has 20 observations. The total sum of squares is 1000 and the model sum of squares is 750. Suppose that you add a third regressor to the model and as a result that model sum of squares is now 785. We are interested in testing that adding this factor has improved the model. What are the degrees of freedom of the test used for this testing?

(a) $1, 16$ ________________ (correct)

(b) $1, 17$

(c) $2, 16$

(d) $3, 16$

(e) $2, 17$

17. Consider the computer output given here. What is the percentage of total variation that is explained by the model?

The regression equation is: $\hat{y} = 253.81 + 2.7738\ x_1 - 3.5753\ x_2$

Predictor	Coef	SE Coef	T	$p-$value
Constant	253.81	4.7810	?	?
X_1	2.7738	0.1846	15.02	?
X_2	-3.5753	0.1526	?	?

$S = 5.05756$ $R^2 = ?$ $R^2_{\text{adj}} = 98.4\%$

Analysis of Variance

Source	*df*	SS	MS	F	$p-$value
Regression	2	22784	11392	?	?
Residual Error	?	?	?		
Total	14	23091			

(a) 98.67% ______________________________ (correct)

(b) 98.40%

(c) 94.94%

(d) 91.72%

(e) 90.06%

18. Consider the computer output given below. Which one of the following statements about the significance of regression is correct at $\alpha = 0.05$?

The regression equation is: $\hat{y} = 242.17 + 2.146\ x_1 - 3.80\ x_2$

Predictor	Coef	SE Coef	T	$p-$value
Constant	242.17	7.55	32.06	?
X_1	2.146	0.895	?	0.034
X_2	3.80	8.51	0.45	?

$S = 9.62073$ $R^2 = 32.64\%$ $R^2_{\text{adj}} = 21.41\%$

Analysis of Variance

Source	*df*	SS	MS	F	$p-$value
Regression	2	538.22	269.11	2.91	?
Residual Error	12	1110.70	92.56		
Total	14	1648.93			

(a) It is insignificant, as $F = 2.91$ which leads to $p-$ value > 0.05 __________ (correct)

(b) It is significant, as $F = 2.91$ which leads to $p-$ value < 0.05

(c) It is significant, as $T = 2.40$ which leads to $p-$ value < 0.05

(d) It is insignificant, as $T = 0.45$ which leads to $p-$ value > 0.05

(e) It is significant, as $T = 32.06$ which leads to $p-$ value < 0.05

19. Consider the computer output given here. What is the value of F-statistic for testing the significance of regression?

The regression equation is: $\hat{y} = 26.753 + 1.4756\ x$

Predictor	Coef	SE Coef	T	$p-$value
Constant	26.753	2.373	?	?
X	1.4756	0.1063	?	?

$S = 2.7004$ $R^2 = 93.7\%$ $R^2_{\text{adj}} = 93.2\%$

Analysis of Variance

Source	*df*	SS	MS	F	$p-$value
Regression	1	?	?	?	?
Residual Error	?	94.8	7.3		
Total	14	1500			

(a) 192.696 ______ (correct)

(b) 1405.2

(c) 7.292

(d) 2.7

(e) 94.8

20. Consider the computer output given here. Compute 98% confidence interval for the slope coefficient β_1. What is the lower limit of the resulting confidence interval?

The regression equation is: $\hat{y} = 26.753 + 1.4756\ x$

Predictor	Coef	SE Coef	T	$p-$value
Constant	26.753	2.373	?	?
X	1.4756	0.1063	?	?

$S = 2.7004$ $R^2 = 93.7\%$ $R^2_{\text{adj}} = 93.2\%$

Analysis of Variance

Source	*df*	SS	MS	F	$p-$value
Regression	1	?	?	?	?
Residual Error	?	94.8	7.3		
Total	14	1500			

(a) 1.1939 ______ (correct)

(b) 20.5263

(c) 1.2476

(d) 1.1591

(e) 1.7545