STAT 416- Major Exam 2

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1 Exercise 1(10 points)

Machine 1 is currently working. Machine 2 will be put in use at a time t from now. If the lifetime of machine i is exponential with rate λ_i, $i = 1, 2$, what is the probability that machine 1 is the first machine to fail?

2 Exercise 2(20=6+7+7 points)

Consider a two-server system in which a customer is served first by server 1, then by server 2, and then departs. The service times at server i are exponential random variables with rates μ_i, $i = 1, 2$. When you arrive, you find server 1 free and two customers at server 2: customer A in service and customer B waiting in line.

1. Find P_A, the probability that A is still in service when you move over to server 2.

2. Find P_B, the probability that B is still in the system when you move over to server 2.

3. Find $E[T]$, where T is the time that you spend in the system.

Hint: Write

$$T = S_1 + S_2 + W_A + W_B$$

where S_i is your service time at server i, W_A is the amount of time you wait in queue while A is being served, and W_B is the amount of time you wait in queue while B is being served.

3 Exercise 3(15=7+8 points)

Let $\{M_i(t), t \geq 0\}$, $i = 1, 2, 3$ be independent Poisson processes with respective rates λ_i, $i = 1, 2, 3$, and set

$$N_1(t) = M_1(t) + M_2(t), \quad N_2(t) = M_2(t) + M_3(t).$$

The stochastic process $\{(N_1(t), N_2(t)), t \geq 0\}$ is called a bivariate Poisson process.

1. Find $P\{N_1(t) = n, N_2(t) = m\}$, where $n \leq m$.

2. Find $\mathrm{Cov}(N_1(t), N_2(t))$.

4 Exercise 4(15=8+7 points)

Customers arrive at a bank at a Poisson rate λ. Suppose two customers arrived during the first hour. What is the probability that:

1. both arrived during the first 20 minutes?

2. at least one arrived during the first 20 minutes?

5 Exercise 5(25=5+5+5+5+10 points)

A job shop consists of three machines and two repairmen. The amount of time a machine works before breaking down is exponentially distributed with parameter λ. If the amount of time it takes a single repairman to fix a machine is exponentially distributed with parameter μ. Let X_t represent the number of machines in use at time t.

1. Derive the generator matrix of $(X_t)_t$.

2. Determine the stationary distribution of $(X_t)_t$.

3. Calculate the average number of machines not in use.

4. Compute the proportion of time during which both repairmen are busy.

5. Assume that initially, two machines are in use. Find the expected amount of time until there is only one machine in use.

6 Exercise 6(15=8+7 points)

A service center consists of two servers, each working at an exponential rate of $\mu = 2$ per hour. If customers arrive at a Poisson rate $\lambda = 3$ per hour, then, assuming a system capacity of at most three customers:

1. What fraction of potential customers enter the system?

2. What would the value of part (a) be if there was only a single server, and their rate was twice as fast (that is, $\mu = 4$)?

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