KING FAHD UNIVERSITY OF PETROLEUM & MINERALS, DHAHRAN, SAUDI ARABIA
DEPARTMENT OF MATHEMATICS

STAT 502: Statistical Inference

Term 211, Second Major Exam, Saturday November 19, 2021, 09:00 AM

Name: ________________________________ ID #: ____________________

Q1: (15 pts.) Suppose that we take a sample of size n_1 from a normally distributed population with mean μ_1 and variance σ_1^2 and an independent of sample size n_2 from a normally distributed population with mean μ_2 and variance σ_2^2. We cannot assume that the unknown variances are equal but we are fortunate enough to know that $\sigma_2^2 = k\sigma_1^2$ for some constant $k \neq 1$. Suppose that the sample means are given by $\bar{X}_1$ and $\bar{X}_2$ and the sample variances by s_1^2 and s_2^2, respectively. Derive a $100(1-\alpha)\%$ confidence interval for $(\mu_1 - \mu_2)$ assuming that $\sigma_2^2 = k\sigma_1^2$ for some constant $k \neq 1$.

Q2: (10 pts.) Let $X_1, X_2, \dots, X_n$ be a random sample from the probability density function given by

$$f(x|\theta) = \begin{cases} \dfrac{\Gamma(2\theta)}{[\Gamma(\theta)]^2} x^{\theta-1}(1-x)^{\theta-1}, & 0 \le x \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the method-of-moments estimator of θ.

$\int_0^1 x^{a-1}(1-x)^{b-1}dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ where $\Gamma(a) = \int_0^\infty x^{a-1}e^{-x}dx$

Q3: (10 pts.) Let $X_1, X_2, \ldots, X_n$ be a random sample from Maxwell distribution with probability density function given by

$$f(x) = \begin{cases} \sqrt{\frac{2}{\pi}} \frac{x^2 e^{-\frac{x^2}{2a^2}}}{a^3}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

where $a > 0$. If the prior distribution of $\theta = \frac{1}{2a^2}$ is Gamma(α, β) i.e. $f(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta}$; $\theta > 0$, find the posterior distribution of θ given sample. Also find the posterior mean which is a Bayesian estimator of θ.

Q4: (8 pts.) Let $X_1, X_2, \ldots, X_n$ denote independent and identically distributed random variables from a power family distribution with parameters $\alpha > 0$ and $\theta > 0$. The density function of X is given as:

$$f\big(x|(\alpha,\theta)\big) = \begin{cases} \dfrac{\alpha x^{\alpha-1}}{\theta^{\alpha}}, & 0 \le x \le \theta \\ 0, & \text{elsewhere.} \end{cases}$$

Derive the method-of-moments estimators of α and θ.

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Good Luck