

**KING FAHD UNIVERSITY OF PETROLEUM & MINERALS, DHAHRAN, SAUDI ARABIA**  
**DEPARTMENT OF MATHEMATICS**

**STAT 502: Statistical Inference**

Term 211, Final Exam, Saturday January 01, 2022, 07:00 PM

Name: \_\_\_\_\_ ID #: \_\_\_\_\_

Q1: (10 points) Let  $p$  be the probability that a coin will fall head in a single toss. In order to test the null hypothesis  $H_0: p = \frac{1}{2}$  against the alternative  $H_1: p = \frac{1}{4}$ , the coin is tossed 50 times. Find the best critical region against fixed probability of type-I error 0.06. Also, find the power of the test.

Q2: (10 points) Let  $f_0(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ ;  $-\infty \leq x \leq \infty$  under  $H_0$  and  $f_1(x) = \frac{1}{2} e^{-|x|}$ ;  $-\infty \leq x \leq \infty$  under  $H_1$ .

For a sample of size  $n$ , show that the best critical region for testing  $H_0$  against  $H_1$  is given by  $\sum_{i=1}^n (|x_i| - 1)^2 \geq k$ .

Let  $n = 1$  and  $k = 1$ , find the probabilities of type-I and type-II errors.

Q3: (10 points) Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from  $N(\mu_X, \sigma^2)$  and  $Y_1, Y_2, Y_3, \dots, Y_m$  be another independent random sample from  $N(\mu_Y, \sigma^2)$ . Develop a statistical test for testing  $H_0: \mu_X = \mu_Y$  against  $H_1: \mu_X \neq \mu_Y$ . Also identify the distribution of test statistic.

Q4: (10 points) Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from  $N(\mu, \sigma^2)$ . Develop a generalized likelihood ratio test (GLRT) for testing  $H_0: \sigma^2 = \sigma_0^2$  against  $H_1: \sigma^2 > \sigma_0^2$ . Also identify the distribution of test statistic.

*Good Luck*