King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia
Department of Mathematics

STAT 502: Statistical Inference

Term 212, Second Major Exam, Monday April 11, 2022, 08:30 PM

Name: ________________________________ ID #: ____________________

Note: Prove every result that you use in your solution.

Q1: Let $\bar{X}$ and $\bar{Y}$ be the means of two independent random samples, each of size n, from the respective distributions $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$, where the common variance is known. Find n such that

$$P\left(\bar{X} - \bar{Y} - \frac{\sigma}{5} < \mu_1 - \mu_2 < \bar{X} - \bar{Y} - \frac{\sigma}{5}\right) = 0.9$$

Q2: Suppose $X_1, X_2, \dots, X_n$ are identically and independently distributed with PDF

$$f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find the maximum likelihood estimator of $P(X \le 2)$.

Q3: Suppose $X_1, X_2, \dots, X_n$ are identically and independently distributed with probability density function $f(x;\mu)$. Consider $\hat{\mu}_1 = g_1(X_1, X_2, \dots, X_n)$ is a sufficient estimator of parameter of μ and $\hat{\mu}_2 = g_2(X_1, X_2, \dots, X_n)$ is an unbiased estimator of μ. Consider another estimator $\hat{\mu}_3 = E(\hat{\mu}_2 \mid \hat{\mu}_1)$ and show that $\hat{\mu}_3$ is unbiased for μ and is never less efficient than $\hat{\mu}_2$.

Q4: Suppose that a random variable X follows a Binomial distribution with probability mass function

$$P(X = x) = \; {}^{k}C_{x}\theta^{x}(1-\theta)^{k-x}, x = 0,1,2, \dots, k$$

where k and θ are the two unknown parameters.

Using a sample of size n from the above-mentioned distribution and derive the method of moments estimators for k and θ.

Q5: Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$ where σ is a known constant i.e.

$$f(x;\mu) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty.$$

Also, suppose that the prior distribution of μ is $N(\theta, \tau^2)$. Derive a Bayesian estimator for μ.

Good Luck