King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia
Department of Mathematics

STAT 502: Statistical Inference

Term 212, Final Exam, Tuesday May 17, 2022, 07:00 PM

Name: ______________________________ ID #: ____________________

Q1: (4 pts.) What is the sufficient statistic for θ if the sample arises from a beta distribution in which $\alpha = \beta = \theta > 0$?

For beta distribution $f(x; \alpha, \beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}$; $0 < x < 1$.

Q2: (8 pts.) Suppose that $Y_1, Y_2, Y_3, \ldots, Y_n$ constitute a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. We want to test $H_0\colon \mu = \mu_0$ versus $H_1\colon \mu > \mu_0$. Find the appropriate likelihood ratio test and recognize the distribution of statistic.

Q2: cont…

Q3: (6 pts.) Let $Y_1, Y_2, Y_3, \ldots, Y_m$ be a random sample from pdf or pmf $f(y; \tau)$ where m is a positive number. Let τ_0 and τ_1 be distinct fixed values of τ and c be a positive number. Let A be a subset of sample space such that

(a) $\frac{L(y;\tau_0)}{L(y;\tau_1)} \leq c$ for each point $\in A$

(b) $\frac{L(y;\tau_0)}{L(y;\tau_1)} \geq c$ for each point $\in A^c$

(c) $\alpha = P_{H_0}[\boldsymbol{Y} \in A]$

Then mathematically show that A is a best critical region of size α for testing the simple hypothesis H_0: $\tau = \tau_0$ against the simple alternative hypothesis H_1: $\tau = \tau_1$.

Q3: cont…

Q4: (4+4 = 8 pts.) Let the independent random variables X and Y have distributions that are $N(\mu_X, 3^2)$ and $N(\mu_Y, 4^2)$, where the means μ_X and μ_Y are unknown. Let $X_1, X_2, X_3, \dots, X_{25}$ and $Y_1, Y_2, Y_3, \dots, Y_{20}$ denote independent random samples from these distributions. For testing a simple null hypothesis H_0: $\mu_X = \mu_Y$ against the simple alternative hypothesis H_1: $\mu_X > \mu_Y$, the critical region of size α is $[\bar{X} - \bar{Y} > 1.77]$.

(a) Find the value of α.

(b) Find the power of this test if $\mu_X = 1.3 + \mu_Y$.

Q4: cont…

Good Luck