KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DEPARTMENT OF MATHEMATICS

STAT 502: Statistical Inference

Term 222, Final Exam

Tuesday May 23, 2023, 07:00 PM

Name: ________________________________ ID #: ____________________

Question No	Full Marks	Marks Obtained
1	**08**	
2	**06**	
3	**08**	
4	**10**	
5	**13**	
6	**05**	
Total	**50**	

Instructions:

1. Mobiles are not allowed in exam. If you have your **mobile** with you, **turn it off** and put it **on the table** so that it is visible to proctor.

2. Show all the derivation steps. There are points for the steps so if your miss them, you lose points. For multiple choice type questions, showing calculation steps is not required.

3. Derive every result that you use in your solution, unless mentioned otherwise.

Q1: (8 pts.) Let $X_1, X_2, \dots, X_n$ denote a random sample from a distribution that has PDF $f(x;\theta)$ where $\theta \in \Omega$. Mathematically prove that the statistic $Y_1 = u_1(X_1, X_2, \dots, X_n)$ is sufficient statistic for θ if and only if we can find two non-negative functions, k_1 and k_2, such that

$$f(x_1;\theta)f(x_2;\theta)\dots f(x_n;\theta) = k_1[u_1(x_1, x_2, \dots, x_n);\theta] \times k_2(x_1, x_2, \dots, x_n),$$

where $k_2(x_1, x_2, \dots, x_n)$ does not depend upon θ.

Q1: cont…

Q2: (6 pts.) Let $X_1, X_2, \dots, X_n$ denote a random sample from a normal distribution with known mean μ_0 and unknown variance $\sigma^2 = \frac{1}{v}$. Further, assuming that the prior distribution for v is gamma with hyperparameters α and β, derive Bayesian estimator for $\sigma^2 = \frac{1}{v}$.

Q2: cont…

Q3: (8 pts.) Let $X_1, X_2, \dots, X_n$ and $Y_1, Y_2, \dots, Y_m$ be independent samples from $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$, respectively where $\sigma_X^2 = c\sigma_Y^2$. Derive a $100(1-\alpha)\%$ confidence interval for the difference between means i.e. $(\mu_X - \mu_Y)$.

Q3: cont…

Q4: (10 pts.) Let $X_1, X_2, \dots, X_n$ denote a random sample from $f(x;\theta)$. The likelihood of $X_1, X_2, \dots, X_n$ is given by $L(x;\theta) = \prod_{i=1}^{n} f(x_i;\theta)$ for $(x_1, x_2, \dots, x_n)$. Let θ' and θ'' be distinct fixed values of θ such that $\Omega = \{\theta: \theta = \theta', \theta''\}$. By defining C as a subset of the sample space S, state and prove Neyman Pearson's lemma.

Q4: cont…

Q5: (6+4+3 = 13 pts.) Let X_1, X_2, … , X_n be a random sample from $\Gamma(\alpha = 3, \beta)$ i.e. $f(x;\beta) = \frac{1}{\Gamma(3)\beta^3} x^2 e^{-\frac{x}{\beta}}; x > 0$.

(a) Derive a likelihood ratio test for testing H$_0$: $\beta = \beta_0$ against H$_1$: $\beta \neq \beta_0$.

(b) Define a pivotal quantity based on your statistic and derive its sampling distribution under null hypothesis.

(c) For testing $\beta = 2$ and $n = 6$, give an expression for the rejection region such that the size of test is 0.05.

Q5: cont…

Q5: cont…

Q6: (5 pts.) Let X_1, X_2, … , X_n be a random sample from the probability density function given by $f(x;\theta) = \frac{\Gamma(2\theta)}{[\Gamma(\theta)]^2} x^{\theta-1}(1-x)^{\theta-1}$, $0 \le x \le 1$ and zero elsewhere. Find the method-of-moments estimator for θ.

[Blank Page I]

[Blank Page II]

[Blank Page III]

Good Luck