King Fahd University of Petroleum and Minerals
Department of Mathematics
STAT 515

MIDTERM EXAM — Term 231
Sunday, October 29, 2023
Location: Building 59, Room: 2011
Allowed Time: 90 minutes
Instructor: Dr. Brahim MEZERDI

Name:

ID #:

Section #:

Instructions:
1. Write clearly and legibly. You may lose points for messy work.
2. Show all your work. No points for answers without justifications !

Question #	Grade	Total Points
1		5
2		5
3		5
4		5
5		5
6		5
Total		30

Exercise 1 (5 points)

Let X_1 and X_2 two independent random variables such that X_1 has a binomial distribution $\mathcal{B}(n_1, p)$ and X_2 has a binomial distribution $\mathcal{B}(n_2, p)$.

1) Find the distribution of the random variable $X_1 + X_2$.

2) Find the conditional ditribution of X_1 given that $X_1 + X_2 = m$.

Solution

Exercice 2 (5 points)
Let X be a normal random variable with parameters μ and σ with density

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2), \ -\infty < x < +\infty$$

1) Let $Y = \exp(X)$. Show that the random variable Y is lognormal with density

$$f_Y(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp(-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2), \ for\ x > 0$$

2) Compute $E(Y)$ and $Var(Y)$. Justify your answer.
Solution

Exercise 3 (5 points)
Let X a random variable with standard gaussian density

$$f_X(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}x^2)$$

i) Compute $E(X)$ and $Var(X)$?
ii) Find the density, the expectation and the vaiance of $Y = 2X + 3$.
Solution

Exercise 4 (5 points)

We throw a balanced coin. The results of the throws are independent random variables $Y_0, Y_1, Y_2, ..., Y_n,$ with values 0 or 1. For $n \geq 1$, we denote

$$X_n = Y_n + Y_{n-1}$$

1) Compute $P(X_3 = 0/X_1 = 0,\ X_2 = 1)$ and $P(X_3 = 0/X_2 = 1)$?

2) Is (X_n) a Markov chain?

Solution

Exercise 5 (5 points)

Consider (Y_n) a sequences of random variables with values in $\mathbb{Z}$, independent and identiquely distributed. Assume that Y_0 is independent of (Y_n). Define the sequence (X_n) by:

$$\begin{cases} X_0 = Y_0 \\ X_{n+1} = X_n + \sum_{i=0}^{n+1} Y_i \end{cases}$$

1) Show that (X_n) is a Markov chain.

2) Assume that for each i the random variable Y_i takes values 1 and -1 with probabilities $P(Y_i = 1) = p$ and $P(Y_i = -1) = 1 - p$ Determine the transition matrix of (X_n).

Exercise 6 (5 points)

Let (X_n) be a Markov chain with finite state space $\{1, 2\}$ such that $P(X_{n+1} = 1/X_n = 1) = 0.7$ and $P(X_{n+1} = 1/X_n = 2) = 0.4$.

1) Determine the transition matrix and draw the transition graph.

2) Find the stationary distribution of the class $\{1, 2\}$.

Solution