KING FAHD UNIVERSITY OF PETROLEUM & MINERALS
DHAHRAN, SAUDI ARABIA
DEPARTMENT OF MATHEMATICS

STAT 530: DESIGN AND ANALYSIS OF EXPERIMENTS

Term 231, Major Exam 1, Saturday October 7, 2023, 12:00PM-3:00PM

Name: ID #:

Instructions:

1. Formula sheet is provided. You are not allowed to bring with you, formula sheet or any other printed/written paper.
2. Mobiles are not allowed in exam. If you have your mobile with you, turn it off and keep it under your seat so that it is visible to proctor. Your mobile(s) should not be in your pocket during the exam.

Question No	Full Marks	Marks Obtained
Part 1	*5*	
Part 2	*40*	
Part 3	*15*	
Total	***60***	

Part 1: **Circle the correct option.**

I. In the ANOVA, the treatment means **(1 mark)**
   a. a factor
   b. blocks
   c. experimental units
   d. different levels of a factor

II. In the design of experiment, blocking is used **(1 mark)**
   a. to reduce bias
   b. to reduce variation
   c. as a substitute for a control group
   d. as a first step in randomization

III. The number of times each experimental condition is observed in a factorial setup is known as
   a. factor **(1 mark)**
   b. replications
   c. treatment.
   d. experimental units

IV. The mean square for error in the ANOVA provides an estimate of
   a. the variance of the random error **(1 mark)**
   b. the variance of an individual treatment average
   c. the standard deviation of an individual observation
   d. none of the above

V. Suppose that a single-factor experiment with five levels of the factor has been conducted. There are three replicates and the experiment has been conducted as a complete randomized design. If the experiment had been conducted in blocks, the pure error degrees of freedom would be reduced by **(1 mark)**
   a. 3
   b. 5
   c. 2
   d. none of the above

Part 2:

1.
   a. Write the statistical model of a single factor experiment for completely randomized design. **(2 marks)**

   b. Given that the sum of square error for a completely randomized design is $SS_E = \sum_{i=1}^{a}\sum_{j=1}^{n}\left(y_{ij} - \bar{y}_{i.}\right)^2$, show that $E(MS_E) = \sigma^2$. **(4 marks)**

c. A single-factor completely randomized design has six levels of the factor. There are five replicates and the total sum of squares is 900.25. The treatment sum of squares is 750.50. What is the estimate of the error variance σ^2? **(4 marks)**

d. The response time in milliseconds was determined for three different types of circuits that could be used in an automatic valve shutoff mechanism. The results from a completely randomized experiment are shown in the following table:

Circuit Type	Response Time				
1	9	12	10	8	15
2	20	21	23	17	30
3	6	5	8	16	7

Assuming non-normality, using an appropriate non-parametric technique to test the hypothesis that the three circuit types have the same response time at α = 0.01. **(5 marks)**

2. A consumer products company relies on direct mail marketing pieces as a major component of its advertising campaigns. The company has three different designs for a new brochure and wants to evaluate their effectiveness, as there are substantial differences in costs between the three designs. The company decides to test the three designs by mailing 5000 samples of each to potential customers in four different regions of the country. Since there are known regional differences in the customer base, regions are considered as blocks. The number of responses to each mailing is as follows

	Region			
Design	**NE**	**NW**	**SE**	**SW**
1	250	350	219	375
2	400	525	390	580
3	275	340	200	310

Analyze the data from this experiment. **(10 marks)**

3. a. Write the statistical model of a single factor experiment for latin square design. Decompose the sum of square total as $SS_T = SS_{Rows} + SS_{Columns} + SS_{Treatments} + SS_E$ **(5 marks)**

b. An industrial engineer is investigating the effect of four assembly methods (A, B, C, D) on the assembly time for a color television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time.

Order of Assembly	Operator			
	1	2	3	4
1	$C = 10$	$D = 14$	$A = 7$	$B = 8$
2	$B = 7$	$C = 18$	$D = 11$	$A = 8$
3	$A = 5$	$B = 10$	$C = 11$	$D = 9$
4	$D = 10$	$A = 10$	$B = 12$	$C = 14$

To account for this source of variability, the engineer uses the Latin square design that follows. Analyze the data from this experiment ($\alpha = 0.05$) and draw appropriate conclusions. **(10 marks)**

Part 3:

Use R/Minitab Software to conduct appropriate analysis to answer the following questions. Be sure to save your important outputs and graph into MSWORD file under your name and email this file to the instructor (jimoh.ajadi@kfupm.edu.sa) at the end of the exam.

The yield of a chemical process was measured using five batches of raw material, five acid concentrations, five standing times (A, B, C, D, E), and five catalyst concentrations (α, β, γ, δ, ϵ). The Graeco-Latin square that follows was used.

a. Analyze the data from this experiment (use α = 0.05) and draw conclusions **(10 marks)**

	Acid Concentration		
Batch	**1**	**2**	**3**
1	$A\alpha = 26$	$B\beta = 16$	$C\gamma = 19$
2	$B\gamma = 18$	$C\delta = 21$	$D\epsilon = 18$
3	$C\epsilon = 20$	$D\alpha = 12$	$E\beta = 16$
4	$D\beta = 15$	$E\gamma = 15$	$A\delta = 22$
5	$E\delta = 10$	$A\epsilon = 24$	$B\alpha = 17$

b. Also, analyze the residuals from this experiment. **(5 marks)**

With Best Wishes

$$SS_{\text{Treatments}} = \frac{1}{p}\sum_{j=1}^{p} y_{.j.}^2 - \frac{y_{...}^2}{N}$$

$$SS_{\text{Rows}} = \frac{1}{p}\sum_{i=1}^{p} y_{i..}^2 - \frac{y_{...}^2}{N}$$

$$H = \frac{1}{S^2}\left[\sum_{i=1}^{a}\frac{R_{i.}^2}{n_i} - \frac{N(N+1)^2}{4}\right]$$

$$S^2 = \frac{1}{N-1}\left[\sum_{i=1}^{a}\sum_{j=1}^{n_i} R_{ij}^2 - \frac{N(N+1)^2}{4}\right]$$

$$SS_{\text{Columns}} = \frac{1}{p}\sum_{k=1}^{p} y_{..k}^2 - \frac{y_{...}^2}{N}$$

$$SS_T = \sum_i\sum_j\sum_k y_{ijk}^2 - \frac{y_{...}^2}{N}$$

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$SS_{\text{Treatments}} = \frac{1}{b}\sum_{i=1}^{a} y_{i.}^2 - \frac{y_{..}^2}{N}$$

$$SS_{\text{Blocks}} = \frac{1}{a}\sum_{j=1}^{b} y_{.j}^2 - \frac{y_{..}^2}{N}$$

$$H = \frac{12}{N(N+1)}\sum_{i=1}^{a}\frac{R_{i.}^2}{n_i} - 3(N+1)$$

no ties