# KING FAHD UNIVERSITY OF PETROLEUM & MINERALS DHAHRAN, SAUDI ARABIA DEPARTMENT OF MATHEMATICS

## **STAT 530: DESIGN AND ANALYSIS OF EXPERIMENTS** Term 231, Major Exam II, Thursday November 16, 2023, 5:00PM-8:00PM

ID #:

### **Instructions:**

- 1. Formula sheet is provided. You are not allowed to bring with you, formula sheet or any other printed/written paper.
- 2. Mobiles are not allowed in exam. If you have your mobile with you, turn it off and keep it under your seat so that it is visible to proctor. Your mobile(s) should not be in your pocket during the exam.

Question No	Full Marks	Marks Obtained
Part 1	5	
Part 2	40	
Part 3	15	
Total	60	

- A factorial experiment may be conducted as a RCBD by running each replicate of the experiment in a unique block.
  (a) True
  - (b) False
- 2. A  $2^3$  factorial is replicated twice. The number of pure error or residual degrees of freedom are
  - (a) 4
  - (b) 12
  - (c) 15
  - (d) 8
  - (e) 2
- 3. In an experimental situation if the same effect is confounded in all the replicates, it is known as a. Fractional factorial
  - b. Partial confounding
  - c. Complete confounding
  - d. Nested design
  - e. Resolution
- 4. Consider the  $2^2$  factorial design in two blocks. If AB is confounded with blocks, then which of the following runs is the same block as *ab*?
  - (a) (1)
  - (b) b
  - (c) a
  - (d) ab
  - (e) both option (b) and (c)
- 5. Suppose that a  $2^2$  design has been conducted. There are four replicates and the experiment has been conducted in four blocks. The error sum of squares is 500 and the block sum of squares is 250. If the experiment had been conducted as a completely randomized design, the estimate of the error variance  $\sigma^2$  would be
  - (a) 25.0
  - (b) 25.5
  - (c) 35.0
  - (d) 38.5
  - (e) none of the above

# Part 2:

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Source	SS	DF	MS	F
A	50.00	1	50.00	?
B	80.00	2	40.00	?
AB	30.00	2	15.00	?
Blocks	10.00	1	?	
Error	?	?	?	
Total	185.00	11		

Consider the following incomplete ANOVA table:

- (a) The pure error estimate of the standard deviation of the sample observations is 1.73.True False (1 mark)
- (b) Suppose that the experiment had not been run in blocks; that is, it is now a CRD. The number of degrees of freedom for error would now be \_\_\_\_\_\_. (1 mark)
- (c) The error mean square in the CRD would be \_\_\_\_\_. (1 mark)

(d) The F-test statistic for interaction in the CRD is significant at  $\alpha = 0.05$ . True False (1 mark)

(e) Write the statistical model of a two-factor factorial experiment using completely randomized design and derive the sum of squares for one of the main effects. (4 marks)

(f) An article in the IEEE Transactions on Electron Devices (Nov. 1986, pp. 1754) describes a study on polysilicon doping. The experiment shown below is a variation of their study. The response variable is base current.

Polysilicon	Anneal Temperature (°C)			
Doping (ions)	900	950	1000	
$1 \times 10^{20}$	4.60	10.15	11.01	
	4.40	10.20	10.58	
$2 \times 10^{20}$	3.20	9.38	10.81	
	3.50	10.02	10.60	

Is there evidence (with  $\alpha = 0.05$ ) indicating that either polysilicon doping level or anneal temperature affects base current? (12 marks)

2. An engineer is interested in the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of a 23 factorial design are run. The results are as follows:

		С	Treatment Combination	Replicate		
A	B			Ι	Π	ш
_	_	_	(1)	22	31	25
+	_	_	а	32	43	29
_	+	_	b	35	34	50
+	+	_	ab	55	47	46
_	_	+	С	44	45	38
+	_	+	ac	40	37	36
_	+	+	bc	60	50	54
+	+	+	abc	39	41	47



a. Estimate the interaction effect of BC



b. Consider the data from the first replicate, suppose that these observations could not all be run using the same bar stock. Set up a design to run these observations in two blocks of four observations each with ABC confounded. Analyze the data (12 marks)

c. Write down a regression model for predicting tool life (in hours) based on the results of this experiment. (4 marks)

#### Part 3:

Use **R Software** to conduct appropriate analysis to answer the following questions. Be sure to save your important outputs and graph into MSWORD file under your name and email this file to the instructor (jimoh.ajadi@kfupm.edu.sa) at the end of the exam.

An article in the AT&T Technical Journal (March/April 1986, Vol. 65, pp. 39–50) describes the application of two-level factorial designs to integrated circuit manufacturing. A basic processing step is to grow an epitaxial layer on polished silicon wafers. The wafers mounted on a susceptor are positioned inside a bell jar, and chemical vapors are introduced. The susceptor is rotated, and heat is applied until the epitaxial layer is thick enough. An experiment was run using two factors: arsenic flow rate (A) and deposition time (B). Four replicates were run, and the epitaxial layer thickness was measured ( $\mu$ m). The data are shown in Table P6.1.

The 2	The 2 <sup>2</sup> Design for Problem 6.16								
		Replicate					Factor	Levels	
<u>A</u>	В	Ι	Π	III	IV		Low (-)	High (+)	
_	_	14.037	16.165	13.972	13.907	A	55%	59%	
+	_	13.880	13.860	14.032	13.914				
-	+	14.821	14.757	14.843	14.878	В	Short	Long	
+	+	14.888	14.921	14.415	14.932		(10 min)	(15 min)	

- a. Conduct an analysis of variance and draw conclusions. Which factors are important? (use  $\alpha = 0.05$ ) (8 marks)
- b. Write down a regression equation that could be used to predict epitaxial layer thickness over the region of arsenic flow rate and deposition time used in this experiment. (2 marks)
- c. Analyze the residuals. Are there any residuals that should cause concern? (5 marks)

With Best Wishes

$$SS_{A} = \frac{1}{bn} \sum_{i=1}^{a} y_{i..}^{2} - \frac{y_{...}^{2}}{abn}$$
$$SS_{B} = \frac{1}{an} \sum_{j=1}^{b} y_{j..}^{2} - \frac{y_{...}^{2}}{abn}$$
$$SS_{Subtotals} = \frac{1}{n} \sum_{i=1}^{a} \sum_{j=1}^{b} y_{ij.}^{2} - \frac{y_{...}^{2}}{abn}$$

$$SS_{AB} = SS_{Subtotals} - SS_A - SS_B$$

$$SS_E = SS_T - SS_{AB} - SS_A - SS_B$$

 $SS_E = SS_T - SS_{\text{Subtotals}}$