STAT565-First Major, Term 222, Date: 02-March-2023

Name: **ID:**

1 (i) Define the following terms:

- Population
- Sample
- Parameter
- Statistic
- Sampling units
- Sampling frame
- Non-sampling error
- Probability sampling
- Non-probability Sampling
- Sampling error

(ii) Show that the sample mean $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ is an unbiased estimator of the population mean, $\bar{Y}$ and its variance is given by $Var(\bar{y}) = \frac{N-1}{N}\frac{S_y^2}{n}$ in with replacement sampling.

(iii) The table below shows the weights at birth of four babies, x delivered in one day in Dammam hospital. Suppose a simple random sample of size $n = 2$ is to be selected from the above table of

U_i	1	2	3	4
X_i	3.9kg	3.6kg	3.4kg	3.7kg

population $N = 4$ units without replacement in order to estimate the population mean weight of the 4 babies. Obtain all possible samples of size 2 selected without replacement together with all possible sample estimates so as to complete the table below.

Sample	Sample units	Probability	Sample mean $\bar{x}$	Square of error $(\bar{x} - \bar{X})^2$	$\frac{1-f}{n(n-1)}\sum_{i=1}^{n}(x_i - \bar{x})^2$
1					
2					
⋮					
Mean					

2 Given the ratio of two variables, $R = \frac{\sum_{=1}^{N} Y_i}{\sum_{i=1}^{N} X_i} = \frac{N\bar{Y}}{N\bar{X}}$ with their corresponding sample estimates

$$\widehat{R} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} = \frac{\bar{y}}{\bar{x}}$$

(i) Find $E(\widehat{R})$ and $Var(\widehat{R})$

(ii) Show that bias of $\widehat{R}$ is defined as $E(\widehat{R}) - R = \frac{1-f}{n} R\left[\frac{S_x^2}{\bar{X}^2} - \frac{S_{xy}}{\bar{X}\bar{Y}}\right]$

3 In a population of size $N = 5$, the values of the main character Y, and the the auxiliary character X, are given as follows:

Y	3	8	4	7	13
X	1	5	3	4	10

Using the above data

(i) Calculate the population mean $\bar{Y}$ of y

(ii) Calculate the population ratio $R = \frac{\bar{Y}}{\bar{X}}$

(iii) Obtain the sample means $\bar{y}$ and $\bar{x}$ for all possible simple random samples of size 3 drawn without replacement.

(iv) Show that $\bar{y}$ is unbiased for $\bar{Y}$ while $\widehat{R}$ is biased for R

(v) Verify that $Var(\bar{y})$, the variances of $\bar{y}$ is given by

$$Var(\bar{y}) = \frac{N - n}{N} \frac{S_Y^2}{n}$$

4 Given a cost function

$$C = C_0 + \sum_{h=1}^{L} C_h n_h \quad \text{and} \quad V(\bar{y}_{st}) = \sum \frac{W_h^2 S_h^2 (1 - f_h)}{n_h}$$

(i) Show that for optimum allocation $n_h = \frac{n W_h S_h / \sqrt{C_h}}{\sum \left(W_h S_h / \sqrt{C_h} \right)}$

(ii) Verify that $V_{prop}(\bar{y}_{st})$ agrees with the formula, $V(\bar{y}_{st}) = \frac{1-f}{n} \sum W_h S_h^2$

(iii) If $V(\widehat{Y}_{st}) = \sum N_h (N_h - n_h) \frac{S_h^2}{n_h}$, show that by putting $n_h = \frac{n N_h}{N}$ in $V(\widehat{Y}_{st})$ gives $V(\widehat{Y}_{st}) = \frac{1-f}{f} \sum N_h S_h^2$